तत् त्वं पूषन अपावृणु

केंद्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN

ZIET CHANDIGARH

WORKBOOK

MATHEMATICS

FOR

CLASS VIII

(PART I)

CONCEPTUAL UNDERSTANDING | PRACTICE REGULARLY | THINK LOGICALLY | SUCCEED CONFIDENTLY

Name of Vidyalaya : _____

Name : _____

Class : _____ Section : _____

Roll No. : _____

संरक्षक

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FOREWORD

Education today aims to develop confident, creative, and competent learners who can apply their knowledge to solve real-life problems. In this context, the implementation of the National Education Policy (NEP) 2020 and the National Curriculum Framework (NCF) 2023 has brought a significant transformation in the teaching and learning of Mathematics. The new NCERT Class VI Mathematics textbook, *Ganita Prakash*, promotes conceptual understanding, logical reasoning, problem-solving, and joyful learning through meaningful experiences.

I am pleased to present the **"Workbook for Students of Class VIII Mathematics (Part I) based on the New NCERT Textbook."** This workbook has been thoughtfully designed to supplement classroom learning and provide students with ample opportunities to practice, explore, and strengthen mathematical concepts.

The workbook contains a variety of competency-based exercises, application-oriented questions, Challenging questions, self-assessment tasks, and real-life situations. These carefully designed learning experiences encourage students to think critically, reason logically, and develop confidence in applying mathematics in everyday life. The exercises also support continuous practice and formative assessment, enabling teachers to monitor students' progress effectively.

I sincerely appreciate the dedicated efforts of the Associate Course Director, Resource Persons and Mathematics faculty who have worked tirelessly to develop this learner-friendly resource. Their commitment towards academic excellence and quality education is truly commendable.

I am confident that this workbook will become a valuable companion for students in their mathematical journey and an effective classroom resource for teachers. I extend my best wishes to all students and teachers for a joyful, engaging, and successful learning experience.

(Deputy Commissioner)
Kendriya Vidyalaya Sangathan
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CONTENT PREPARED BY:

TOPIC	NAME OF TEACHER (TGT Maths)	NAME OF KV	REGION
A SQUARE AND A CUBE	KALPNA GUPTA	SUBATHU	GURUGRAM
POWER PLAY			
A STORY OF NUMBERS	NEELAM YADAV	PALWAL	GURUGRAM
QUADRILATERALS			
NUMBER PLAY	PRIYANKA SHARMA	DHARAMSHALA	GURUGRAM
WE DISTRIBUTE, YET THINGS MULTIPLY			
PROPORTIONAL REASONING-1	GARIMA BHATT	ITBP BHANU	GURUGRAM

MODERATED BY:

TOPIC	NAME OF TEACHER (TGT Maths)	NAME OF KV	REGION
A SQUARE AND A CUBE	DEEPAK KUMAR	SECTOR-22 ROHINI	DELHI
POWER PLAY	YOGITA KHANNA	SHAKURBASTI	DELHI
A STORY OF NUMBERS	NITESH KUMAR	DR. RAJENDRA PRASAD	DELHI
QUADRILATERALS	ANNU	NO.1 DELHI CANTT.	DELHI
NUMBER PLAY	NEERU RANI	GOLE MARKET	DELHI
WE DISTRIBUTE, YET THINGS MULTIPLY	OM PARKASH SINGH	BHIKHIWIND	CHANDIGARH
PROPORTIONAL REASONING-1			

1 A SQUARE AND A CUBE

Numbers reveal beautiful patterns. Squares and cubes are two such special patterns.

IN THIS CHAPTER YOU WILL LEARN

- Square numbers and their properties
- Square roots
- Cube numbers and their properties
- Cube roots
- Patterns, estimates and number sense
- Historical connections and puzzles

THE LOCKER PUZZLE (Chapter Opening)

100 lockers numbered 1 to 100 are toggled in rounds.

- Person 1 opens every locker.
- Person 2 toggles every 2nd locker.
- Person 3 toggles every 3rd locker … and so on.

A locker is toggled as many times as the number of factors of its number. Only lockers with an odd number of factors remain open — i.e., the square numbers.

1	2	…	10
11	12	…	20
…	…	4	…
91	92	…	100

Open lockers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

The passcode was hidden in the lockers touched exactly twice — the prime numbers: 2 – 3 – 5 – 7 – 11.

KEY DEFINITIONS

- **Square Number:** A number obtained by multiplying a number by itself. $n \times n = n^2$ (Read as 'n squared')
- **Perfect Square:** The squares of natural numbers.
- **Square Root:** If $y = x^2$, then x is the square root of y. Denoted by $\sqrt{y}$ (positive square root). Every perfect square has two integral square roots: $+\sqrt{y}$ and $-\sqrt{y}$.
- **Cube (or Cube Number):** A number obtained by multiplying a number by itself three times. $n \times n \times n = n^3$ (Read as 'n cubed')
- **Perfect Cube:** The cubes of natural numbers.
- **Cube Root:** If $y = x^3$, then x is the cube root of y. Denoted by $\sqrt[3]{y}$.

SQUARE NUMBERS

Squares of first 10 natural numbers

n	n^2	Value	n	n^2	Value
1	1^2	1	6	6^2	36
2	2^2	4	7	7^2	49
3	3^2	9	8	8^2	64
4	4^2	16	9	9^2	81
5	5^2	25	10	10^2	100

Properties & Patterns

- Perfect squares end with 0, 1, 4, 5, 6 or 9.
- Squares can have only an even number of zeros at the end.
- A number ends in 2, 3, 7 or 8 → Not a square.
- If a number has 1 or 9 in the units place, its square ends in 1.
- If a number has 6 in the units place, its square also ends in 6.
- Squares increase by consecutive odd numbers.

SQUARES AS SUM OF ODD NUMBERS

$1 = 1$
$4 = 1 + 3$
$9 = 1 + 3 + 5$
$16 = 1 + 3 + 5 + 7$
$25 = 1 + 3 + 5 + 7 + 9$
$36 = 1 + 3 + 5 + 7 + 9 + 11$

In general: n^2 = sum of first n odd numbers.

FACTORS AND SQUARES

- A locker is toggled as many times as the number of factors of its number.
- Numbers with an odd number of factors are exactly the square numbers.
- Every factor (except in a square number) has a partner factor. In a square number, one factor pairs with itself.

6 : 1×6, 2×3	4 : 1×4, 2×2	9 : 1×9, 3×3	1 : 1×1
Factors: 1, 2, 3, 6 (Even) Closed	Factors: 1, 2, 4 (Odd) Open	Factors: 1, 3, 9 (Odd) Open	Factors: 1 (Odd) Open

Only square numbers remain open.
Hence, open lockers are the squares.

FINDING SQUARE ROOTS

Methods

1. List squares in order.
2. Use: n^2 = sum of first n odd numbers. Subtract successive odd numbers.
3. Prime factorisation: factors in pairs.
4. Estimation:
   - Find nearest perfect squares.
   - Check units digit (√ ends in 0,1,2,3,4,5,6,7,8,9).

Examples

- $\sqrt{1936} = 44$ (since $44^2 = 1936$)
- $\sqrt{576} = 24$ (since $576 = (2^2)(3^2)(4^2) = (2\times3\times4)^2$)
- $40 < \sqrt{1936} < 50$, last digit 4 or 6
  $45^2 = 2025 > 1936 \Rightarrow \sqrt{1936} = 44$

CUBE NUMBERS

Cubes of first 10 natural numbers

n	n^3	Value	n	n^3	Value
1	1^3	1	6	6^3	216
2	2^3	8	7	7^3	343
3	3^3	27	8	8^3	512
4	4^3	64	9	9^3	729
5	5^3	125	10	10^3	1000

Properties & Patterns

- Possible last digits: 0, 1, 8, 9, or even number of zeros.
- A cube cannot end with exactly two zeros.
- Number of digits in cubes:
  1–9 (1 digit), 10–21 (2 digits), 22–46 (3 digits), …
- n^3 increases by adding $(3n^2 + 3n + 1)$ each time.

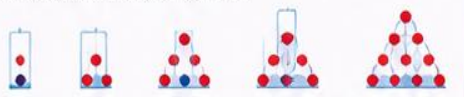
CUBES AS SUM OF CONSECUTIVE ODD NUMBERS

$1 = 1$
$8 = 1 + 3 + 5$
$27 = 1 + 3 + 5 + 7 + 9$
$64 = 1 + 3 + 5 + 7 + 9 + 11 + 13$
$125 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$

In general: n^3 = sum of first n^2 odd numbers.

MORE ABOUT SQUARES

- The difference between consecutive squares is an odd number.
- Numbers between n^2 and $(n+1)^2$ are:
  $n^2 + 1, n^2 + 2, \ldots, (n+1)^2 - 1$ → Total $2n$ numbers.
- There are n perfect squares between $(n-1)^2$ and n^2.
  e.g., between 0 – 100: 1–10 squares, 100 – 200: 11–14 squares, 200 – 300: 15–17 squares, …
- Relation with triangular numbers:



1 = 1^2 | 1+3=4 = 2^2 | 1+3+5=9 = 3^2 | 1+3+5+7=16 = 4^2 | 1+3+5+7+9=25 = 5^2

EXAMPLES OF PATTERNS

1. Units digit pattern (Squares)
   Possible last digits: 0, 1, 4, 5, 6, 9.
   Not possible: 2, 3, 7, 8.
2. Zeros at the end
   If a number ends with k zeros, its square ends with $2k$ zeros.
3. Parity
   Even² = Even, Odd² = Odd.
4. Example sets
   $1^2, 9^2, 11^2, 19^2, 21^2, 29^2, \ldots$ end in 1.
   $4^2, 6^2, 16^2, 26^2, \ldots$ end in 6.

FINDING CUBE ROOTS

Methods

1. Prime factorisation: factors in triplets.
2. List cubes in order.
3. Using pattern & estimation.

Example

- $3375 = 3\times3\times3\times5\times5\times5 = (3\times5)^3 = 15^3$
  $\Rightarrow \sqrt[3]{3375} = 15$
- Estimation:
  $15^3 = 3375 < x < 16^3 = 4096 \Rightarrow \sqrt[3]{3500} \approx 15$.

SQUARE & CUBE COMPARISON

Aspect	Square (n^2)	Cube (n^3)
Definition	$n \times n$	$n \times n \times n$
Notation	n^2	n^3
Roots	$\sqrt{n}$	$\sqrt[3]{n}$
Factors (for perfect)	In pairs	In triplets
Unit digit pattern	0,1,4,5,6,9	0,1,8,9 or even 0's
Zeros rule	k zeros → $2k$ zeros	k zeros → $3k$ zeros
Sum of odd numbers	First n odd numbers	First n^2 odd numbers

HISTORICAL CONNECTION

- **Babylonians** (1700 BCE) compiled lists of squares and cubes for practical use in land measurement and construction.
- **In ancient India,** squares were called "varga" and cubes were called "ghana".
- **Aryabhata** (499 CE) defined varga as both the square figure and the square of a number.
- The word *mula* (root) from Sanskrit (meaning root of a plant, basis, origin) gave rise to the words *jidhr* (Arabic) and *radix* (Latin) — from which we get "root" in mathematics.
- **Ramanujan** (1910s) astonished Hardy by identifying 1729 as the smallest number that is the sum of two cubes in two different ways: $1^3 + 12^3 = 9^3 + 10^3 = 1729$ (Taxicab number).

PUZZLE CORNER

1. Square Pairs!

Arrange numbers 1 to 17 in a row so that the sum of every adjacent pair is a square. (Hint: unique arrangement exists.)

2. Arrange numbers 1 to 32 in a circle so that adjacent numbers add up to a square.

IMPORTANT FORMULAS

- $n^2 = n \times n$
- $\sqrt{n^2} = n$ (for positive n)
- $n^3 = n \times n \times n$
- $\sqrt[3]{n^3} = n$
- $n^2 - (n-1)^2 = 2n - 1$ (odd number)
- $n^3 - (n-1)^3 = 3n^2 - 3n + 1$
- Sum of first n odd numbers $= n^2$
- Sum of first n^2 odd numbers $= n^3$

KEY TAKEAWAYS

- ✓ Squares and cubes have beautiful patterns.
- ✓ Square numbers are sums of first n odd numbers.
- ✓ Cube numbers are sums of first n^2 odd numbers.
- ✓ Prime factorisation helps identify perfect squares (pairs) and perfect cubes (triplets).
- ✓ Square root (√) and cube root ($\sqrt[3]{}$) are inverse operations.
- ✓ Patterns help us estimate and solve large problems.

REMEMBER

Look for patterns, factorise smartly, estimate wisely, and numbers will reveal their secrets!

A SQUARE AND A CUBE

WORKSHEET-1

Section A: MCQs (Part-1) (10 × 1 = 10 marks)

1. The total non-square numbers between the squares of 30 and 31 are

(a) 60 (b) 61 (c) 62 (d) 63

2. If a number has 2 or 2 at its unit's place, then its square would end in

(a) 2 (b) 8 (c) 4 (d) 16

3. How many perfect square numbers lie between 16 and 81?

(a) 4 (b) 5 (c) 6 (d) 8

4. Which number is called the Hardy-Ramanujan number?

(a) 1839 (b) 1729 (c) 1949 (d) 1529

5. Cube root of 8 is

(a) 512 (b) 64 (c) 2 (d) 4

6. Which of the following numbers cannot be a perfect square?

(a) 2500 (b) 1600 (c) 961 (d) 468

7. Which of the following numbers would not have the digit 4 at its unit place?

(a) 52^2 (b) 68^2 (c) 34^2 (d) 102^2

8. The partner factors of 25 is/are

(a) 1, 25 (b) 5, 5 (c) 2, 25 (d) Both (a) and (b)

9.A number ending in 9 will have the unit place of its square as:

(a) 1 (b) 3 (c) 9 (d) 6

10.Which of the following is not a unit digit of a square number?

(a) 0 (b) 4 (c) 6 (d) 8

MCQs (Part-2) (10 × 1 = 10 marks)

11.The square root of 0.000025 is:

(a) 0.005 (b) 0.0005 (c) 0.025 (d) 0.00025 2.

12.How many natural numbers lie between 52 and 62?

(a) 9 (b) 10 (c) 11 (d) 12 3.

13. Find the number whose square is 7569:

(a) 87 (b) 86 (c) 88 (d) 89

14.A square board has an area of 144 square units. How long is each side of the board?

(a) 11 units (b) 12 units (c) 13 units (d) 14 units 5.

15.If one member of a Pythagorean triplet is 2m, then the other two members are:

(a) m, $m^2 + 1$ (b) $m^2 + 1$, $m^2 – 1$ (c) m^2, $m^2 – 1$ (d) m^2, $m + 1$

16. A number has the prime factorisation $\mathbf{2^3 \times 3^2 \times 5^2}$. What is the **smallest number** by which it should be multiplied to make it a perfect square?

(a) 2 (b) 3 (c) 5 (d) 6

17. Which of the following numbers is a **perfect cube but not a perfect square**?

(a) 64 (b) 125 (c) 729 (d) 144

18. The difference between the squares of two consecutive natural numbers is **41**. What is the smaller number?

(a) 19 (b) 20 (c) 21 (d) 22

19. Which of the following numbers has an **integer square root** as well as an **integer cube root**?

(a) 324 (b) 729 (c) 4096 (d) 1000

20. A cube-shaped box has a volume of $\mathbf{2744\ cm^3}$. What is the length of each edge of the box?

(a) 12 cm (b) 13 cm (c) 14 cm (d) 16 cm

WORKSHEET-2

Section B: Short Answer (10 × 2 =20 marks)

1. Find the smallest square number which is exactly divisible by 12, 15, and 20.

2. How many non-perfect square numbers lie between the squares of 12 and 13?

3. Is 160 a perfect square? Why or why not?

4. Find the unit digit of the cube of 234.

5. Roxie has 7 dresses, 2 hats, and 3 pairs of shoes. How many different ways can Roxie dress up?

6. Find the smallest number by which 250 must be divided so that the quotient becomes a perfect cube?

7. Find the square root of 2401 by prime factorization method.

8. Find the cube root of 2744, by the prime factorization method.

9. Write 49 as a sum of odd natural numbers.

10. Find the smallest number to multiply by 1800 to make it a perfect square.

Section C: Short Answer (6 × 3 = 18 marks)

11. What would the thickness of a sheet of paper be after 30 folds? Assume that the thickness of the sheet is 0.001 cm.

12. There are 5 bottles in a container. Every day, a new container is brought in. How many bottles would be there after 40 days?

13. . Find the smallest number by which 200 must be multiplied so that the product becomes a perfect cube.

14. Find the smallest number by which 882 must be multiplied to make it a perfect square.

15. Find the square root of 4356, i.e., evaluate $\sqrt{4356}$ by prime factorisation.

16. Find the side of a cubical box whose volume is 1728 cm^3.

Section D: Long Answer (4 × 5 = 20 marks)

17. A school has 1475 students. What is the minimum number of students that need to be added so that the total number of students can be arranged in a perfect square grid?

18. A school has 50 lockers numbered 1 to 50. Students perform the following steps:

- Student 1 opens all lockers.
- Student 2 toggles (opens/closes) every 2nd locker (2,4, 6,…).
- This continues until Student 50 toggles the 50th locker, in the last.

- Which lockers remain open at the end? Explain why.

19. Numbers between the squares of the following pairs: (i) 25 and 26 (ii) 77 and 78

20. By which should we multiply 675 to make it a perfect cube? Also, find the cube root of the perfect cube so obtained.

Case Study Based Questions (2 × 4 = 8 marks)

21. Case Study – **The Stones that Shine ...**

Three daughters with curious eyes, Each got three baskets — a kingly prize. Each basket had three silver keys, Each opens three big rooms with ease. Each room had tables — one, two, three, With three bright necklaces on each, you see. Each necklace had three diamonds so fine…

(i) Can you count these stones that shine?

OR

Can you count the number of tables?

(ii) How many rooms were there altogether?

(iii) Write the product p4 × p6 in exponential form.

22. Math club discussion: During a math club discussion, Arjun told the famous story of Hardy and Ramanujan. Hardy thought 1729 was dull, but Ramanujan replied it was special since $1729 = 1^3 + 12^3 = 9^3 + 10^3$. This made Arjun curious about cubes and their properties. He started exploring other such numbers called "taxicab numbers."

Q1. Find the units digit of the cube of 1729.

Q2. What is special about taxi cab numbers ?

Q3. What is the cube root of 1729?

OR

Can a perfect cube end with the digits 00? Justify your answer.

Self Assessment:

I understood all concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood the all the concepts ☐ Yes ☐ No

He/She need more practice ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment:

Answers of Worksheet-1:

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans	a	c	a	b	c	d	a	d	a	d
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans	a	a	a	b	b	a	b	b	c	b

Answers of Worksheet-2:

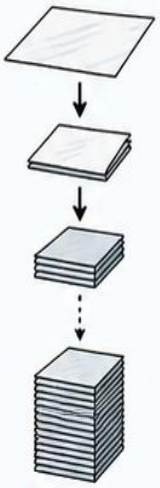
Q.No	1	2	3	4	5	6	7	8	9	10
Ans	900	24	No	4	42way	3	49	14	49	2
Q.No	11	12	13	14	15	16	17	18	19	20
Ans	10.74km	200 bottles	5	2	66	12cm	46	1,4,9, 16,25, 36,49	(i)50 (ii)154	15
Q.No	21			22						
Ans	2187 OR 243, 81 P^{10}			9, It is smallest number that can be written as the sum oftwo cubes in two different ways. 12.0023 Or no						

2 POWER PLAY

Just 46 folds of a paper can reach the Moon!

1. THE IDEA

- Fold a sheet of paper. Each time you fold, its thickness doubles.
- This is an example of exponential (multiplicative) growth.
- Amazing fact: After just 46 folds, the thickness is more than 7,00,000 km — enough to reach the Moon!



2. THICKNESS AFTER FOLDS

Initial thickness of paper = 0.001 cm

Thickness doubles after each fold.

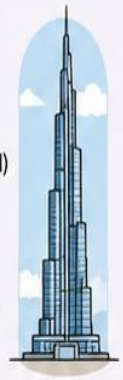
Fold	Thickness	Fold	Thickness	Fold	Thickness
1	0.002 cm	7	0.128 cm	13	8.192 cm
2	0.004 cm	8	0.256 cm	14	16.384 cm
3	0.008 cm	9	0.512 cm	15	32.768 cm
4	0.016 cm	10	1.024 cm	16	65.536 cm
5	0.032 cm	11	2.048 cm	17	≈ 131 cm
6	0.064 cm	12	4.096 cm		

(We use '≈' to mean approximately equal to.)

After 10 folds ≈ **1.024 cm** | After **17 folds** ≈ **131 cm** (a little more than 4 feet)

3. AMAZING GROWTH!

Folds	Approx. Thickness	
After 26 folds	≈ 670 m	(Burj Khalifa is 830 m tall)
After 30 folds	≈ 10.7 km	(Height of planes fly)
After 46 folds	> 7,00,000 km	(Can reach the Moon!)



4. THICKNESS DOUBLE EVERY TIME!

From any point, the thickness after 10 more folds increases by 10^{24} times.

Range of Folds	Thickness (approx.)	Times Increased By
0 to 10	1.023 cm	10^{24}
10 to 20	10.474 m	10^{24}
20 to 30	10.726 km	10^{24}
30 to 40	10984.2 km	10^{24}

5. EXPONENTIAL NOTATION – KEY CONCEPTS

General Form

n^a means n multiplied by itself a times.

Example: $5^4 = 5 \times 5 \times 5 \times 5 = 625$

Base = 5 Exponent (power) = 4

Common Powers

$n^2 = n \times n$ (read as 'n squared')

$n^3 = n \times n \times n$ (read as 'n **cubed**')

$n^4 = n \times n \times n \times n$ (read as 'n to the power 4')

$n^7 = n \times n \times n \times n \times n \times n \times n$'

(read as 'n to the power 7')

and so on...

Examples

- $4^3 = 4 \times 4 \times 4 = 64$
- $(-4)^3 = (-4) \times (-4) \times (-4) = -64$
- $a \times a \times a \times b \times b = a^3b^2$
- $a \times a \times b \times b \times b \times b = a^2b^4$

Remember!

$4 + 4 + 4 = 3 \times 4 = 12$

but

$4 \times 4 \times 4 = 4^3 = 64$

Express as Product of Prime Factors

Example: Express 32400 in exponential form.

$32400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3$

$= 2^4 \times 5^2 \times 3^4$

Try It!

- What is $(-1)^5$? → −1 (negative)
- What is $(-1)^{56}$? → +1 (positive)
- Is $(-2)^4 = 16$? → Yes

6. QUICK FORMULA

If the initial thickness of paper is v cm, then thickness after n folds = $v \times 2^n$

Example: If $v = 0.001$ cm and $n = 10$, then thickness = $0.001 \times 2^{10} = 1.024$ cm

Key Takeaway

Small steps of doubling **create huge results** – that's the **Power Play!**

POWER PLAY

WORKSHEET-1

Section A: MCQs (/Part-1) (10 × 1 = 10 marks)

1.. 81 can be expressed as:
(a) 9^2 (b) 3^4 (c) 27^3 (d) 3^3

2. Which is a perfect sixth power?
(a) 729 (b) 64 (c) 729^6 (d) 1

3. The value of 2^4 is
(a) 8 (b) 4 (c) 16 (d) 32

4. What is the value of $3^0 + 4^0 + 5^0$?

(a) 3 (b) 1 (c) 12 (d) 0 2.

5. The planet Mars is approximately 22, 80, 00,000 km away from the Sun. What is this distance in standard form?
(a)22.8×107 km (b) 2.28×108 km (c) 2.28×107 km (d) 2.28×10−8 km .

6. The value of $(-2)^3 \times (-2)^{-5}$ is:

(a) 2^2 (b) -1/4 (c) −4 (d) 4^4.

7. The scientific notation of 51.26000 is:
(a) 5.126×10^4 (b) 5.126×10^1 (c) 5.126×10^{-4} (d) 5.126×10^{-6}

8. Simplify the expression: $(p^5 \times q^6) / (p^2 \times q^3)$

(a) $p^3 q^3$ (b) $(pq)^7$ (c) $p^7 q^9$ (d) p^3 / q^3 .

9. A virus's DNA is 0.0000000021 meters long. In scientific notation, this length is:

(a) 2.1×10−9 m (b) 2.1×10−8 m (c) 2.1×109 m (d) 2.1×108 m .

10. A magic potion doubles its quantity every minute. If you start with a single drop, how many minutes will it take for the quantity to be 2 15 drops?

(a) 15 minutes (b) 30 minutes (c) 10 minutes (d) 2 minutes

MCQs (/Part-2) (10 × 1 = 10 marks)

11. The hypotenuse of a right triangle with its legs of lengths 3x and 4x is
(a)5x (b) 7x (c) 16x (d) 25x 7.

12. The square of 12.5 is:
(a) 156.25 (b) 125.25 (c) 162.25 (d) 152.25

13. The value of 5^2 x 5^{-2} is
(a) 1 (b) 5 (c) 5^4 (d) 5^{-4}

14. A lock uses 4 slots with digits 0-9. How many combinations are possible?
(a) 10^3 (b) 10^4 (c) 4^{10} (d) 9^4

15. If there are 3×10^{12} trees and each tree has about 10^4 leaves, what is the total number of leaves?
(a) 3×10^{16} (b)3×10^8 (c) 30×10^{12} (d) 10^{18}

16. A bacteria culture doubles every hour. If there are 8 bacteria initially, how many bacteria will be present after 5 hours?
(a) 128 (b) 256 (c) 160 (d) 320

17. Which of the following is equal to $5^8 \div 5^3$?
(a) 5^{24} (b) 5^5 (c) 5^{11} (d) 5^{-5}

18. A city has a population of 8,450,000. Which of the following represents this number in scientific notation?
(a) 84.5×10^5 (b) 8.45×10^6 (c) 0.845×10^7 (d) 845×10^4

19. Simplify $(3^4)^2$.
(a) 3^6 (b) 3^8 (c) 6^4 (d) 9^4

20. A digital lock has 4 positions, and each position can be filled with any one of 10 digits (0–9). How many different passwords are possible?
(a) 4,000 (b) 10,000 (c) 40,000 (d) 1,000

WORKSHEET-2

Section B: Short Answer (10 × 2 =20 marks)

1. Express the following in exponential form:
(i) $5 \times 5 \times 7 \times 7 \times 7$ (ii) $a \times a \times a \times c \times c \times c \times c \times d$

2. Simplify and express the answer in exponential form: $(-4)^5 \text{ X } (-4)^{-10}$

3. Express $(81)^{-2}$ as a power with base 3.

4. What is $(-1)^5$? Is it positive or negative? What about $(-1)^{56}$?

5.Find the value of 126^2 , if $125^2 = 15625$.

6. Express the following as a product of powers of their prime factors in exponential form. -540

7. What is $2^{100} \div 2^{25}$ in powers of 2?

8. Simplify and write the answers in exponential form.

(i) $2^{-4} \times 2^{7}$ (ii) $3^{2} \times 3^{-5} \times 3^{6}$

9. Express the following numbers in standard form. (Page 32)
(i) 59,853 (ii) 65,950

10. If you have lived for a million seconds, how old would you be?

Section C: Short Answer (6 × 3 = 18 marks)

11. Write the numerical value of each of the following: $(-2)^{5} \times (-10)^{6}$

12. If $3x = 9^{2} \times 2^{7}$, find the value of x.

13. A dairy plans to produce 8.5 billion packets of milk in a year. They want a unique ID (identifier) code for each packet. If they choose to use the digits 0-9, how many digits should the code consist of?

14. Estu and Roxie came across a safe containing old stamps and coins that their great-grandfather had collected. It was secured with a 5-digit password.

(i) How many passwords did they end up checking?
(ii) If we have a 2-digit lock and try to find out how many passwords are possible.

15. Find the value of each of the following. $8^3 - 7^3$

16. The students of class 8 of a school donated ₹ 1296 to PM's relief fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

Section D: Long Answer (4 × 5 = 20 marks)

17. Find the value of (i) $(3^2 \times 3^4) \div 3^6$ (ii) $(5^8 \div 5^3) \times 5^2$

18. Numbers between the squares of the following pairs:
(i) 25 and 26 (ii) 77 and 78

19. Find the greatest value among the following. Give a reason.
(i) $74^3 - 73^3$ (ii) $66^3 - 65^3$ (iii) $74^2 - 73^2$ (iv) $66^2 - 65^2$

20. Evaluate $\sqrt[3]{1331} + \sqrt[3]{2744} + \sqrt[3]{1728}$

Case Study Based Questions (2 × 4 = 8 marks)

21. The King's Gold Coins: A king distributes his gold coins to his three ministers. Each minister receives three large chests. Each chest contains three medium-sized boxes. Each medium-sized box contains three small pouches, and each small pouch contains three gold coins. The king asks his ministers to calculate the total number of gold coins, but they must use exponential notation to present their answer

Q1. Express the number of gold coins each minister receives in exponential form.

Q2. How many gold coins were distributed in total by the king? Express your answer in exponential form.
Q3. If the number of ministers was four instead of three, and the number of chests, boxes, pouches, and coins per pouch remained the same, how many total coins would have been distributed? Express your answer in exponential form.
Q4**.** The king takes back a portion of the total coins equal to 3 4 . What is the number of coins remaining with the ministers? Express your answer in exponential form.

22. The Virus Spread: A new virus strain is discovered that infects people at an alarming rate. Initially, there is only one person infected on Day 0. The number of infected people doubles every day. The government wants to track the spread of this virus using exponential notation to manage resources efficiently.

Self Assessment:

I understood all concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood the all the concepts ☐ Yes ☐ No

He/She need more practice ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment

Answers of Worksheet-1:

Q	1	2	3	4	5	6	7	8	9	10
Ans	b		c	a	b	b	b	a	b	a
Q	11	12	13	14	15	16	17	18	19	20
Ans	a	a	a	b	a	b	b	b	b	b

Answers of Worksheet-2:

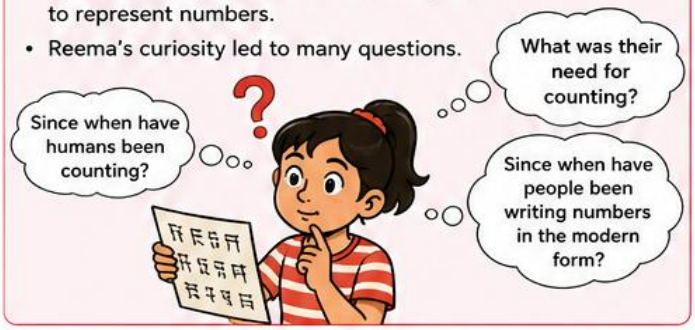
Q	1	2	3	4	5	6
Ans	(i)52x73 ii)a3xc4xd	$(-4)^{-5}$	(3)-8	-1,1	15876	$-(2^2x3^3x5^1)$
Q	7	8	9	10	11	12
Ans	2^{75}	2^2,3^3	(i)$5.9853x10^4$ (ii)$6.595x10^4$	11.57days	-32000000	3,456
Q	13	14	15	16	17	18
Ans	10digits	(i)100000 (ii)100	169	36	(i)1 (ii)78125	(i)50 (ii)154
Q	19	20	21		22	
Ans	(i)	37	3^4, 3^5, $4x3^4$		2^5, 8, 27	

3 A STORY OF NUMBERS

From ancient symbols to modern numbers – a journey through time!

1. REEMA'S CURIOSITY

- Reema found a paper with strange symbols.
- Her father explained that around 4000 years ago, the Mesopotamians used such symbols to represent numbers.
- Reema's curiosity led to many questions.



2. WHY DID HUMANS COUNT?

- Even in the Stone Age, humans needed to count.
- They counted food, animals, goods in trade, offerings in rituals, and the passing of days.
- They wanted to predict events like new moon, full moon, and seasons.
- But their ways of writing numbers were different from what we use today.

3. ORIGIN OF OUR MODERN NUMBER SYSTEM

Origin in India

- Ancient Indian texts like the *Yajurrveda Samhita* used names of numbers based on powers of 10.
- Example: *eka* (1), *dasha* (10), *shata* (100), *sahasra* (1,000), *āyuta* (10,000)... up to 10^{12} and beyond.

Early use of 0 and ten digits

- The first known use of numbers with ten digits (including 0 as a dot) is in the Bakhshali manuscript (c. 3rd century CE).
- Aryabhata (c. 499 CE) explained and used this system in his works.

Spread to the Arab world

- Around 800 CE, the Indian system reached the Arab world.
- Popularised by Al-Khwārizmī in *On the Calculation with Hindu Numerals* (c. 825) and Al-Kindi in *On the Use of the Hindu Numerals* (c. 830).

Reach to Europe and the world

- Reached Europe and parts of Africa by around 1100 CE.
- Fibonacci (c. 1200) encouraged Europe to adopt Indian numerals.
- Widely accepted in Europe by the 17th century.
- Now used in every corner of the world!

Why 'Arabic' or 'Hindu' numerals?

- Europeans called them 'Arabic numerals' as they learned them from the Arab world.
- Arabs called them 'Hindu numerals'.
- Today we use terms like 'Hindu numerals', 'Indian numerals', or 'Hindu-Arabic numerals'.

KEY TAKEAWAY

Our modern number system has its root in India and travelled across the world to become the most efficient way to write and work with numbers.

4. EVOLUTION OF DIGITS (0 TO 9)

Stage/Period	0	1	2	3	4	5	6	7	8	9
Brahmi (c. 3rd century BCE)	•	𑁒	𑁓	𑁔	𑁕	𑁖	𑁗	𑁘	𑁙	𑁚
Hindu (Gwalior) (c. 6th century)	०	१	२	३	४	५	६	७	८	९
Sanskrit–Devanagari (c. 8th–10th century)	०	१	२	३	४	५	६	७	८	९
West Arabic (Gubar) (c. 10th century)	•	١	٢	٣	٤	٥	٦	٧	٨	٩
East Arabic (c. 10th century)	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
11th century (Apices)	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
15th century	o	1	2	3	4	5	6	7	8	9
16th century (Dürer)	0	1	2	3	4	5	6	7	8	9

Over centuries, these shapes evolved into the digits we use today.

5. IMPORTANCE

- The Indian numeral system made calculations simpler and faster.
- It laid the foundation for mathematics, science and technology.
- Its global adoption shows the power of human knowledge and sharing.

6. QUICK FACTS

- Counting began: Stone Age
- Mesopotamian symbols used: Around 4000 years ago
- First use of 0 in Bakhshali manuscript: c. 3rd century CE
- Indian system reached Arabs: Around 800 CE
- Reached Europe: Around 1100 CE (Popularised by Fibonacci c. 1200)
- Widely accepted in Europe: By the 17th century

Numbers tell the story of human thinking, curiosity and innovation across time and civilisations!

STORY OF NUMBERS

WORKSHEET-1

PART –I Section A: MCQs (10 × 1 = 10 marks)

1. Which of the following is the Hindu-Arabic numeral for the Roman numeral 'XLIV'?

(A)44 (B)46 (C)54 (D)64

2. The Egyptian numeral system was based on which number as its base?

(A)12 (B)20 (C)10 (D)60

3. In the Roman numeral system, the symbol 'M' represents:

(A)100 (B)500 (C)1000 (D)5000

4. Which ancient number system used a base-60 (sexagesimal) system?

(A) Egyptian (B) Mayan (C) Roman (D) Mesopotamian

5. The number zero (0) as a placeholder was first formally developed by which civilisation?

(A) Egyptian (B) Roman (C) Indian (Hindu) (D) Mayan

6. In the Roman system, 'IV' means:

(A)4 (B)6 (C)4 (D)1

7. Which of the following number systems is additive (values are simply added)?

(A) Hindu-Arabic (B) Egyptian hieroglyphic

(C) Mayan (D) Mesopotamian cuneiform

8. The Mayan number system was based on which number as its primary base?

(A)10 (B)12 (C)20 (D)60

9. The place value system (positional notation) is a key feature of which number systems?

(A) Roman and Egyptian (B) Hindu-Arabic and Mesopotamian

(C) Roman and Mayan (D) Egyptian and Chinese

10. How is the number 2024 written in Roman numerals?

(A) MMXXIV (B) MMXXVI (C) MCMXXIV (D) MMCDXXIV

MCQs (PART-2) (10 × 1 = 10 marks)

11. In the Mesopotamian system, which two symbols were used to write all numbers?

(A) | and < (B) | and Y (C) O and X (D) | and —

12. Which number system used shells (zero), dots (ones), and bars (fives) as its symbols?

(A) Chinese (B) Egyptian (C) Mayan (D) Roman

13. In the traditional Chinese rod numeral system, numbers are written:

(A) Left to right only (B) Bottom to top only

(C) In vertical columns, top to bottom (D) As hieroglyphs

14. The Egyptian numeral for 1,000 was represented by:

(A) A lotus flower (B) A coiled rope (C) A pointing finger (D) A tadpole

15. In the Mesopotamian base-60 system, how would 70 be written?

(A) As 1 group of 60 + 10 (B) As 7 tens

(C) As 14 fives (D) As 3 groups of 20 + 10

16. Which of the following correctly expresses the Mayan number for 33?

(A) One bar + three dots in the ones place

(B) One dot (top row) + one bar + three dots (bottom row)

(C) Three bars + three dots

(D) One bar + three dots across two rows

17. The Hindu-Arabic numeral system uses how many symbols (digits)?

(A)8 (B)9 (C)10 (D)12

18. What is special about the number system developed in ancient India that changed mathematics worldwide?

(A) It used symbols for only odd numbers

(B) It introduced place value and the digit zero

(C) It was based on 60

(D) It used only addition

19. In the Roman numeral system, which rule applies when a smaller numeral appears BEFORE a larger one?

(A) It is added (B) It is multiplied (C) It is subtracted (D) It is ignored

20. An archaeologist finds a tablet with the symbol 'III IIII' in Babylonian. In our numeral system this represents 3 × 60 + 4 = ?

(A)34 (B)180 (C)184 (D)204

WORKSHEET-1

Section B: Short Answer (10 × 2 = 20 marks)

1. Convert the Roman numeral MCMXCIX into Hindu-Arabic numerals. Show each step.

2. Write the number 487 in Egyptian hieroglyphic notation. Describe each symbol used.

3. In the Mayan number system, represent the number 38. Draw and label each symbol.

4. The Mesopotamians used base-60. Write 125 in Mesopotamian notation (as groups of 60s, 10s, and 1s). What would it look like in cuneiform symbols?

5. Why is the Hindu-Arabic number system considered superior to the Roman numeral system? Give TWO specific reasons.

6. Convert the number MMMDCCXLVIII (Roman) to Hindu-Arabic. Show each symbol's value.

7. The Chinese counting rod system alternates vertical and horizontal rods. Write the number 352 using rod numerals description (vertical for hundreds, horizontal for tens, vertical for ones).

8. An Egyptian scribe writes a number using: one 'finger' (10,000), three 'lotus flowers' (1,000 each), and four 'coiled ropes' (100 each). What is this number?

9. In the Mayan system, what is the place value of the THIRD row (counting from bottom)? What number does one dot in that position represent?

10. State ONE key difference between the Egyptian number system and the Hindu-Arabic number system with a suitable example.

Section C: Short Answer (6 × 3 = 18 marks)

11. Compare the Roman and Egyptian number systems under the following heads:

(a) Base used (b) Presence of zero (c) Use of place value (d) Ease of arithmetic.

Which system do you think was more practical for trade? Give a reason.

12. Ravi has ₹3,749 in his piggy bank. He wants to write this amount in FOUR different ancient number systems.

(a) Roman numerals (b) Egyptian hieroglyphics (describe symbols)

(c) Mayan system (rows and symbols) (d) Explain why this would be easier in Hindu-Arabic.

13. The Babylonians used base-60. Our modern system of measuring time (60 seconds = 1 minute, 60 minutes = 1 hour) is inherited from them.

(a) How many seconds are in 2 hours 35 minutes using Babylonian logic?

(b) Express 9,345 seconds as hours, minutes and seconds.

(c) Why do you think base-60 was convenient for this purpose?

14. Examine the Mayan number system:

(a) Draw and identify all three Mayan symbols and their values.

(b) Represent the number 97 in the Mayan system (show row-wise).

(c) Add the Mayan numbers for 18 and 25 and express the result in both Mayan and Hindu-Arabic.

15. The concept of ZERO is one of India's greatest gifts to the world.

(a) Which ancient Indian mathematician formally introduced the rules of zero?

(b) Why was the introduction of zero as a number (not just a placeholder) a revolutionary idea?

(c) Give TWO examples to show how arithmetic would be difficult without zero.

16. Investigate the Chinese number system:

(a) How does the traditional Chinese written numeral system express the number 2,356?

(b) Compare the Chinese number system with the Hindu-Arabic system — which uses place value?

(c) Write 45,678 in Chinese numeral system notation (describe each component).

Section D: Long Answer (3 × 5 = 15 marks)

17. A historian finds an ancient Roman inscription on a monument:
"Anno Domini MCMLXXXIX" (meaning 'In the Year of the Lord...')

(a) Convert MCMLXXXIX to Hindu-Arabic numerals. Show each numeral's value.

(b) What year does this represent? What significant event could this year mark?

(c) Write the current year (2025) in Roman numerals.

(d) How many years have passed between these two dates? Write this difference in Roman numerals.

(e) Explain TWO limitations of the Roman numeral system that make such calculations difficult.

18. A museum exhibit compares five ancient number systems by representing ONE number — 365 (days in a year).

(a) Express 365 in the Roman numeral system.

(b) Express 365 in the Egyptian hieroglyphic system (describe each symbol).

(c) Express 365 in the Mesopotamian (base-60) system. Show: 365 = ? × 60 + ? × 1.

(d) Express 365 in the Mayan vigesimal (base-20) system. Show: 365 = ? × 20 + ?.

(e) Which system do you think makes it easiest to represent 365? Justify.

19. Investigate how the development of the Hindu number system changed the world:

(a) Name the key contributions: the digits 0-9, place value, decimal point.

(b) Show how addition of 4,567 + 3,894 is performed in Hindu-Arabic vs. how complex it would be in Roman numerals.

(c) Explain the concept of place value with an example: Why does the digit '3' in 3,000 represent a different value from '3' in 30?

(d) How did Indian mathematicians (Brahmagupta, Aryabhata) influence world mathematics?

(e) Name TWO modern technologies or fields that would NOT be possible without the Hindu number system.

SECTION-E Case Study Based Questions (2 × 4 = 8 marks)

20. The Museum of Ancient Civilisations

Class VIII students visit a Museum of Ancient Civilisations. They see a timeline wall that shows how different cultures wrote numbers. One exhibit features an Egyptian papyrus (c. 1650 BCE) showing the number 4,352 written as: four lotus flowers (1,000 each), three coiled ropes (100 each), five heel bones (10 each), and two strokes (1 each). Another display shows the same number 4,352 in Roman numerals on a stone inscription. The guide explains: 'The Egyptians and Romans both lacked a symbol for zero and did not use place value. This made multiplication and division very hard for these civilisations!' A student asks: 'How would the Mesopotamians or the Hindus have written 4,352?'

20(i). Verify: does the Egyptian papyrus correctly represent 4,352? Show the calculation. Also write 4,352 in Roman numerals and identify any challenge in doing so.

20(ii). How would the Hindus (using the Hindu-Arabic system) write 4,352? Explain the role of EACH digit's place value. Why is the Hindu system easier for multiplication than the Egyptian or Roman system?

21. **Decoding an Ancient Mayan Calendar**

An archaeologist uncovers a Mayan stone tablet. It records two dates: Date A is written as two rows — top row: one dot; bottom row: three bars and two dots. Date B is written as three rows — top row: two dots; middle row: one bar; bottom row: three dots. The Mayan calendar uses a base-20 system where the bottom row represents ones (0–19), the second row represents 20s, and the third row represents 360s (not 400s, because of how their calendar works — one row = 360 for time purposes). Each dot = 1, each bar = 5, a shell = 0. The archaeologist needs to decode these date numbers into our Hindu-Arabic system to identify the time gap between them

Q31(i). Decode Date A and Date B from Mayan notation into Hindu-Arabic numerals. Show full working for each. (Use: Row 1 value × 360 + Row 2 value × 20 + Row 3 value.)

Q31(ii). Find the difference between Date B and Date A (in Hindu-Arabic). What does this number of days roughly equal in years (use 1 year ≈ 365 days)? Why do you think the Mayans developed such a sophisticated calendar system?

Self Assessment:

I understood all concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood the all the concepts ☐ Yes ☐ No

He/She need more practice ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment

Answer of Worksheet-1

Q.NO.	1	2	3	4	5	6	7	8	9	10
ANS.	A	C	C	D	C	A	B	C	B	A
Q.NO.	11	12	13	14	15	16	17	18	19	20
ANS.	A	C	C	A	A	B	C	B	C	C

Answer of Worksheet-2

Q.NO.	1	2
ANS.	1999	4 'lotus flowers' + 8 'heel bones' + 7 'strokes'
Q.NO.	3	4
ANS.	Row 2 (top) = 1 dot (×20=20); Row 1 (bottom) = 3 bars + 3 dots (=18). Total = 20+18 = 38	2 groups of 60 + 5 ones → '2, 5' in base-60
Q.NO.	5	6
ANS.	Any Two reasons	3748
Q.NO.	7	8
ANS.	3 vertical rods (hundreds) | 5 horizontal rods (tens) | 2 vertical rods (ones)	13,400

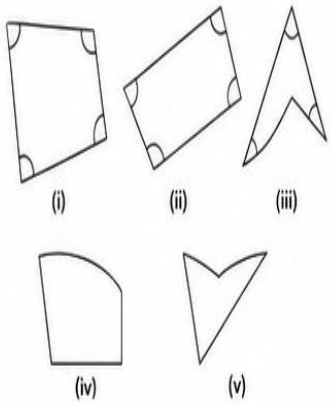
Q.NO.	9	10
ANS.	360 (for calendar) or 400 (for pure base-20)	Egyptian is additive (no place value); Hindu-Arabic is positional (place value).
Q.NO.	11	12
ANS.	Comparison in own words	₹3,749 in four systems
Q.NO.	13	14
ANS.	Babylonian base-60 and time	a) Three Mayan symbols: Shell (•) = 0; Dot (•) = 1; Bar (—) = 5. (b) 97 in Mayan: 97 = 4×20 + 17. Top row (20s): 4 dots. Bottom row (1s): 3 bars(=15) + 2 dots(=2) = 17. Total: 80+17=97 ✓ (c) 18 Mayan: 0 in top, 1 bar+3 dots (=18) bottom. 25 Mayan: 1 dot top (=20), 1 bar (=5) bottom. Sum = 18+25=43. 43=2×20+3: top row=2 dots, bottom=3 dots. Roman: XLIII.
Q.NO.	15	16
ANS.	(a) Brahmagupta. (b) Zero as a NUMBER (not just a space) enabled: place value systems to work correctly; algebra; calculus; the concept of negative numbers; and modern computing (binary 0 and 1).	(a) 2,356 in Chinese: 二千三百五十六 (er qian san bai wu shi liu): 2×1000 + 3×100 + 5×10 + 6×1. The Chinese system names each digit with its place value word. (b) Chinese vs Hindu-Arabic: Both use a form of place value (multiplier × place). Hindu-Arabic is purely positional (position alone gives value); Chinese uses explicit place-name words (thousand, hundred, ten). (c) 45,678: 四万五千六百七十八 = 4×10,000 + 5×1,000 + 6×100 + 7×10 + 8×1
Q.NO.	17	18
ANS.	(a) M=1000, CM=900, L=50, XXX=30, IX=9. Total: 1000+900+50+30+9 = 1989. (b) 1989 = the year the Berlin Wall fell (a historic event). (c) 2025 in Roman: MM=2000, XX=20, V=5 → MMXXV. (d) 2025−1989 = 36 years. 36 in Roman = XXXVI.	(a) Roman: CCCLXV (C=100×3=300, L=50, X=10, V=5 → 300+50+10+5=365). (b) Egyptian: 3 coiled ropes (3×100=300) + 6 heel bones (6×10=60) + 5 strokes (5×1=5) = 365. (8 individual symbols.) (c) Mesopotamian base-60: 365 = 6×60 + 5×1 = 360+5. Written as '6 5' in cuneiform (six tens-wedge groups + five unit-wedges). (d) Mayan base-20: 365 = 18×20 + 5. Bottom row (ones): 1 bar (=5). Top row (20s): 18 = 3 bars + 3 dots. (Note: this is pure base-20; calendar uses 360 base.)
Q.NO.	19	20
ANS.	(a) Key contributions: 10 digits (0–9), place value (positional notation), decimal fractions, concept of zero as a number and placeholder. (b) Addition: 4567+3894 = 8461 (carried out in seconds). In Roman: MMMMDLXVII + MMMDCCCXCIV — no standard algorithm; must convert! Roman multiplication requires lengthy lookup tables (like the Salamis tablet).	i) Egyptian papyrus is correct: 4×1000+3×100+5×10+2 = 4352 ✓. Roman: MMMMCCCLII Egyptian verification: 4 lotus flowers=4×1000=4000; 3 coiled ropes=3×100=300; 5 heel bones=5×10=50; 2 strokes=2. Total=4000+300+50+2=4352 Roman: 4352 = MMMMCCCLII (four M's, three C's, L=50, two I's). Challenge: Roman has no standard single numeral for 4000 — writing four M's (MMMM)

	(c) Place value: 3,000 — the digit 3 is in the THOUSANDS place, so it represents 3×1000=3000. 30 — the digit 3 is in the TENS place, representing 3×10=30. Same symbol, different position = completely different value. This is the power of the Hindu system. (d) Brahmagupta (628 CE): defined rules for +,-,×,÷ with zero and negative numbers. Aryabhata (499 CE): used zero implicitly in astronomical calculations. Their work spread to the Arab world (Al-Khwarizmi) → Europe	is cumbersome; in some traditions 4000=MV̄ (bar notation). This shows Roman's limitation with large numbers. (ii).Hindu-Arabic: 4,352 — digit 4 (thousands)=4000, digit 3 (hundreds)=300, digit 5 (tens)=50, digit 2 (ones)=2 Place value breakdown: 4352 = 4×1000 + 3×100 + 5×10 + 2×1.
Q.NO.	21	
ANS.	**(i).** Date A: Top row (×360): 1 dot = 1×360 = 360? NO — given system: Row1×360 + Row2×20 + Row3×1. top row = one dot = 1; second row = three bars+two dots = 15+2=17. No third row given. Date A = 1×20 + (3 bars+2 dots) = 20 + 17 = 37? Let's use the case's own rule: 2 rows given for Date A. Actually: Top row = 1 dot (=1×360? or ×20). Case says Row1=360s. If 2 rows: row1=20s, row2=ones (simpler). Date A (2 rows): Row 1 (×20)=1 dot=20. Row 2 (×1)=3 bars+2 dots=17. Date A=20+17=37. Date B (3 rows): Row 1 (×360)=2 dots=720. Row 2 (×20)=1 bar=5×20=100. Row 3 (×1)=3 dots=3. Date B=720+100+3=823. **(ii)**Using above: 823 − 37 = 786 days difference. 786 ÷ 365 ≈ 2.15 years (approximately 2 years and 56 days). The Mayans developed a sophisticated calendar because: they were expert astronomers who tracked Venus, the Moon and the Sun with remarkable accuracy; their agricultural society depended on knowing planting and harvest seasons precisely; and their religious ceremonies were tied to celestial cycles.	

4 QUADRILATERALS — SUMMARY OF CONCEPTS & DEFINITIONS

1. QUADRILATERAL – BASICS

- A quadrilateral is a four-sided polygon.
- The angles of a quadrilateral are the angles between its sides.
- Figs. (i), (ii) and (iii) are quadrilaterals. The others are not.



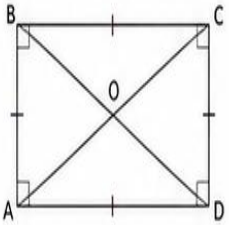
Quadrilateral = 4 sides, 4 vertices, 4 angles

2. RECTANGLE

Definition: A rectangle is a quadrilateral in which–

(i) All angles are right angles (90°).

(ii) Opposite sides are equal.



Properties (using diagonals)

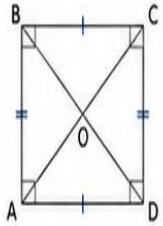
- Diagonals are equal. (AC = BD)
- Diagonals bisect each other. (OA = OC and OB = OD)
- Diagonals make any angle with these properties.
- Opposite sides are equal.
- All angles are 90°.

3. SQUARE

Definition: A square is a quadrilateral in which–

(i) All angles are right angles (90°), and

(ii) All sides are of equal length.

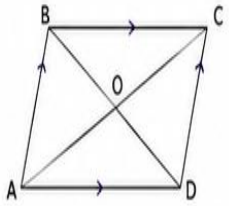


Properties

- All properties of a rectangle.
- All sides are equal (AB = BC = CD = DA).
- Diagonals are equal and bisect each other at right angles.
- Each diagonal bisects the angles of the square.

4. PARALLELOGRAM

Definition: A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.

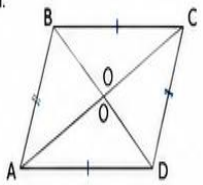


Properties (using diagonals)

- Opposite sides are equal and parallel.
- Opposite angles are equal.
- Consecutive (adjacent) angles are supplementary (add up to 180°).
- Diagonals bisect each other. (OA = OC, OB = OD)
- Diagonals are not necessarily equal.

5. RHOMBUS

Definition: A rhombus is a quadrilateral in which all four sides are of equal length.

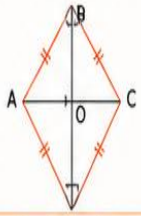


Properties (using diagonals)

- All sides are equal (AB = BC = CD = DA).
- Opposite sides are parallel.
- Opposite angles are equal.
- Consecutive angles are supplementary.
- Diagonals bisect each other.
- Diagonals are perpendicular to each other.
- Each diagonal bisects the angles of the rhombus.

6. KITE

Definition: A kite is a quadrilateral in which two pairs of adjacent sides are equal.

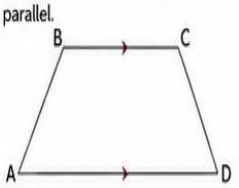


Properties (using diagonals)

- Two pairs of adjacent sides are equal.
- One pair of opposite angles is equal (∠A = ∠C).
- The diagonals are perpendicular to each other.
- One diagonal bisects the other.
- The diagonal along the axis of symmetry (BD) bisects the angles at B and D.

7. TRAPEZIUM (Trapezoid)

Definition: A trapezium is a quadrilateral in which only one pair of opposite sides are parallel.



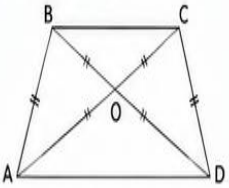
Properties

- Exactly one pair of opposite sides is parallel.
- The other two sides are non-parallel.
- No special relation between angles and diagonals.

Note: If the non-parallel sides are equal, it is called an isosceles trapezium. Then, the base angles are equal and diagonals are equal.

8. ISOSCELES TRAPEZIUM

Definition: A trapezium in which the non-parallel sides are equal.



Properties

- One pair of opposite sides is parallel.
- Non-parallel sides are equal.
- Base angles are equal (∠A = ∠D, ∠B = ∠C).
- Diagonals are equal (AC = BD).
- Consecutive angles on the same leg are supplementary.

9. QUICK COMPARISON TABLE

Figure	Definition (Key Condition)	Opposite Sides	Opposite Angles	Consecutive Angles	Diagonals	Special Property
Rectangle	4 right angles; opposite sides equal	Equal	Equal	Supp.	Equal; bisect each other	All angles 90°
Square	4 right angles; all sides equal	Equal	Equal	Supp.	Equal; bisect at 90°	All sides equal
Parallelogram	Both pairs opposite sides parallel	Equal & //	Equal	Supp.	Bisect each other	Diagonals not necessarily equal
Rhombus	All sides equal	Equal & //	Equal	Supp.	Bisect; ⊥ to each other	Diagonals bisect angles
Kite	Two pairs adjacent sides equal	–	One pair equal	–	⊥ to each other; one bisects the other	One diagonal bisects angles
Trapezium	One pair of opposite sides parallel	–	–	–	No special relation	–
Isosceles Trapezium	One pair parallel; non-parallel sides equal	–	Base angles equal	Same leg supp.	Diagonals are equal	–

10. KEY TAKEAWAYS

- A quadrilateral has 4 sides, 4 vertices and 4 angles.
- Rectangles have 4 right angles and opposite sides equal.
- Squares have all properties of rectangles + all sides equal.
- Parallelograms have both pairs of opposite sides parallel.
- Rhombuses have all sides equal.
- Kites have two pairs of adjacent sides equal.
- Trapeziums have one pair of opposite sides parallel.
- Isosceles trapeziums have equal non-parallel sides and equal base angles.
- Diagonals help in identifying and proving properties.

Think! Diagonals help us reveal the hidden relationships in quadrilaterals!

QUADRILATERAL

WORKSHEET-1

Section A: MCQs(PART –I) (10 × 1 = 10 marks)

1. A simple closed figure made of only line segments is called a:

(A)Curve (B) Polygon (C) Circle (D) Ray

2. The sum of interior angles of a triangle is:

(A) 90° (B) 180° (C) 270° (D) 360°

3. Which of the following is NOT a quadrilateral?

(A) Square (B) Rectangle (C) Pentagon (D) Rhombus

4. A quadrilateral has _____ diagonals.

(A) 1 (B) 2 (C) 3 (D) 4

5. Two angles of a quadrilateral are 70° each and a third is 110°. The fourth angle is:

(A) 100° (B) 110° (C) 120° (D) 130°

6. A parallelogram in which all sides are equal is called a:

(A) Rectangle (B) Trapezium (C) Rhombus (D) Square

7. In a rectangle, each interior angle measures:

(A) 60° (B) 90° (C) 120° (D) 450°

8. Which of the following statements is TRUE about a square?

(A) Opposite sides are unequal (B) Diagonals are perpendicular bisectors of each other

(C) Only one pair of parallel sides (D) Diagonals are unequal

9. The number of sides of a polygon whose interior angle sum is 1080° is:

(A) 6 (B) 7 (C) 8 (D) 9

10. Which of the following pairs of angles are supplementary?

(A) 45° and 55° (B) 60° and 130° (C) 70° and 110° (D) 80° and 90°

MCQs(PART-2) (10 × 1 = 10 marks)

11. The sum of all interior angles of a quadrilateral is always:

(A) 180° (B) 270° (C) 360° (D) 540°

12. In parallelogram ABCD, if ∠A = 65°, then ∠C = ?

(A) 115° (B) 65° (C) 90° (D) 125°

13. Which property is EXCLUSIVE to a rectangle (not common to all parallelograms)?

(A) Opposite sides are equal (B) Diagonals bisect each other

(C) Each angle is 90° (D) Opposite angles are equal

14. In a parallelogram, adjacent angles are:

(A) Equal (B) Supplementary (C) Complementary (D) Vertically opposite

15. A quadrilateral that has exactly ONE pair of parallel sides is called a:

(A) Parallelogram (B) Rhombus (C) Trapezium (D) Kite

16. The diagonals of a rhombus bisect each other at:

(A) 45° (B) 60° (C) 90° (D) 180°

17. ABCD is a parallelogram. If AB = 7 cm and BC = 5 cm, the perimeter is:

(A) 12 cm (B) 24 cm (C) 35 cm (D) 22 cm

18. Which of the following is NOT a property of a parallelogram?

(A) Opposite sides are parallel (B)) Diagonals are equal in length

(C) Opposite angles are equal (D) Consecutive angles are supplementary

19. If the diagonals of a quadrilateral bisect each other at right angles and all sides are equal, the quadrilateral is a:

(A) Rectangle (B) Rhombus (C) Square (D) Trapezium

20. In a quadrilateral PQRS, ∠P + ∠Q = 200°. Then ∠R + ∠S = ?

(A) 160° (B) 180° (C) 200° (D) 360°

Section B: Short Answer (10 × 2 = 20 marks)

1. Find the value of x if the four angles of a quadrilateral are x°, (x + 20)°, (x + 30)° and (x + 50)°.

2. Two adjacent angles of a parallelogram are in the ratio 2:7. Find all four angles of the parallelogram.

3. The perimeter of a rhombus is 52 cm. Find the length of each side.

4. In parallelogram ABCD, ∠A = 3x + 10° and ∠B = 5x – 30°. Find the value of x and both angles.

5. Name the type of quadrilateral formed if one pair of opposite sides is parallel and non-equal, and the other pair is non-parallel. Give one real-life example.

6. In a parallelogram PQRS, the diagonals intersect at O. If PO = 4.5 cm, find PR. Also find SQ if SO = 3.2 cm.

7. State whether the following is True or False with a reason: 'Every rhombus is a parallelogram, but every parallelogram is not a rhombus.'

8. A quadrilateral has angles in the ratio 1:2:3:4. Find each angle and name the type of quadrilateral (if any special type applies).

9. Define a kite. How many pairs of equal adjacent sides does a kite have? Draw a rough labelled sketch.

10. The length of one diagonal of a rhombus is 16 cm and the area is 96 cm^2. Find the length of the other diagonal.

11. In quadrilateral ABCD, $\angle A = 90°$, $\angle B = (2x + 15)°$, $\angle C = (3x – 5)°$ and $\angle D = (x + 10)°$.
(a) Find x. (b) Find all four angles. (c) Is ABCD a parallelogram? Justify.

12. Prove that the diagonal of a parallelogram divides it into two congruent triangles.
Use parallelogram ABCD with diagonal AC. State the properties used at each step.

13. ABCD is a rhombus. The diagonals AC and BD intersect at O. If AC = 24 cm and BD = 10 cm, (a) find all sides of the rhombus, (b) find its perimeter, and (c) find its area.

14. A garden in the shape of a parallelogram has base 45 m and the corresponding height 20 m.
(a) Find the area of the garden.
(b) If fencing costs ₹35 per metre and the adjacent side of the parallelogram is 30 m, find the cost of fencing the garden.

15. Show with a counter-example that the statement 'If a quadrilateral has two pairs of equal opposite angles, it must be a parallelogram' is NOT always true. Also state when it IS true.

16. The angles of a quadrilateral are $(3y - 5)°$, $(y + 15)°$, $(2y + 10)°$ and $(4y - 20)°$.
(a) Find y. (b) Find each angle. (c) Classify the quadrilateral based on its angles.

Section D: Long Answer (3 × 5 = 15 marks)

17. ABCD is a parallelogram. E is the midpoint of AB and F is the midpoint of CD.
(a) Prove that AEFD is a parallelogram.
(b) Prove that EBCF is also a parallelogram.
(c) What can you conclude about the area of each smaller parallelogram compared to ABCD?
(d) If ABCD has area 120 cm^2, what is the area of AEFD?

18. A farmer has a quadrilateral plot PQRS where PQ || SR. PQ = 80 m, SR = 50 m, and the perpendicular distance between PQ and SR is 30 m.

(a) Name this quadrilateral and state two of its properties.

(b) Find the area of the plot.

(c) The farmer wants to divide the plot into two triangles by drawing diagonal PR. If PR = 58 m and the height from S to PR is 20 m, find the area of triangle PRS.

(d) Find the area of triangle PQR. Does the sum match the total area?

19. Investigate: Under what conditions does a parallelogram become

(i) a rectangle, (ii) a rhombus, and (iii) a square?

(a) State the additional condition for each.

(b) Draw and label all three figures.

(c) Write ANY two properties that are common to all three.

(d) A quadrilateral has diagonals that are equal in length AND bisect each other. What type of quadrilateral must it be? Give a reason.

Case Study Based Questions (4 × 2 = 8 marks)

20.The School Playground

The school authority decides to redesign the playground into a parallelogram-shaped field ABCD. The construction team measures AB = 60 m, BC = 40 m, and ∠DAB = 70°. The playground is to be divided into two zones by drawing diagonal BD. The head teacher also wants to install corner boundary posts and a fence along all four sides. She notices that the diagonals of the parallelogram bisect each other at point O, and OA = 37 m.

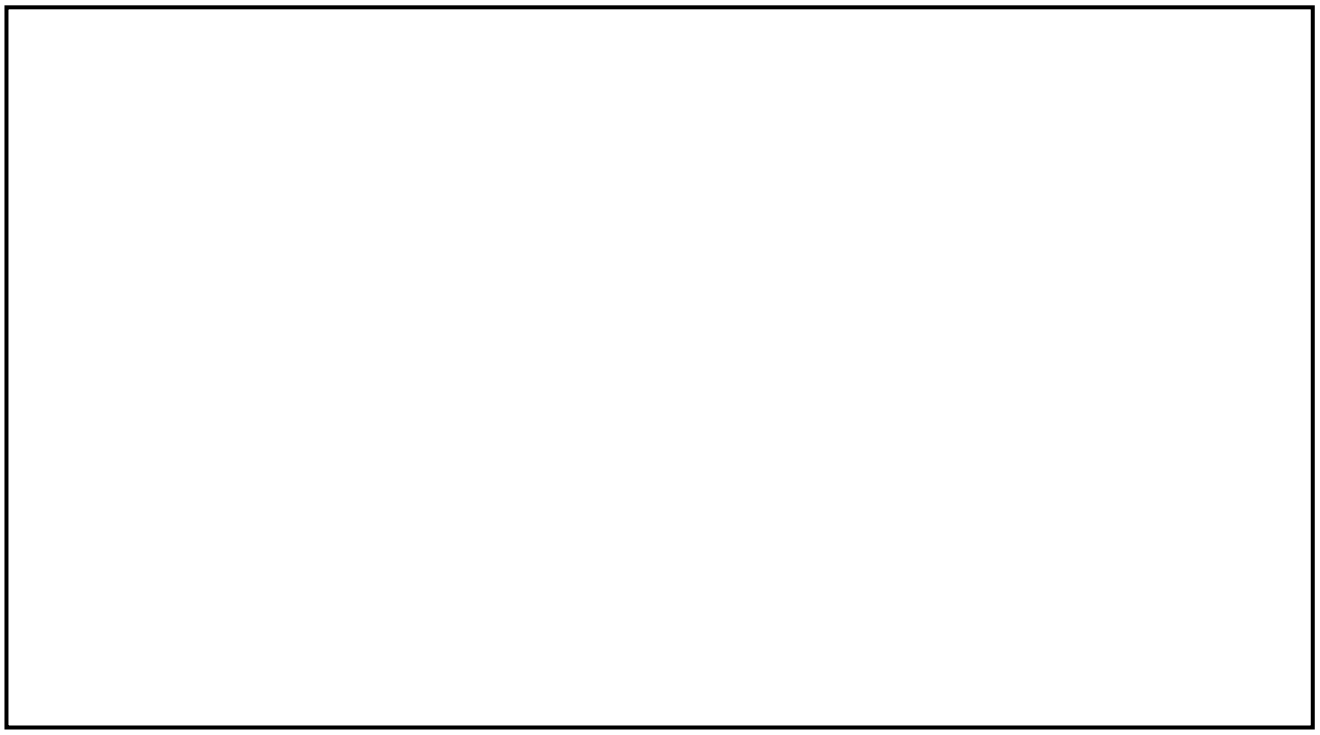
20(i). Find ∠BCD and ∠ABC in the parallelogram ABCD. How are consecutive angles in a parallelogram related? Calculate the total cost of fencing if the rate is ₹50 per metre.

(ii). If OA = 37 m, find the full length of diagonal AC. Also, the diagonal BD = 46 m. Using these diagonals, Ravi claims he can find the area of each triangle formed. Find the area of triangle ABD if its base AB = 60 m and the height from D to AB is 24 m.

21. An interior designer is creating a mosaic floor pattern using rhombus-shaped tiles. Each tile has a perimeter of 48 cm. The diagonals of each tile are in the ratio 3:4. The longer diagonal of one tile is 24 cm. The designer also notices that when four such rhombus tiles are arranged around a common centre point, the angles at the centre always sum to exactly 360° — a property she uses to ensure the tiles fit seamlessly without gaps. She wants to calculate the amount of material needed for 150 such tiles.

21(i). Find the side of each rhombus tile and the length of the shorter diagonal. Using the diagonals, find the area of one tile and verify using the given ratio.

(ii). Find the total area of material needed for 150 tiles. Also, at the centre where four tiles meet, if each tile contributes an angle of 90°, what type of quadrilateral is formed at the centre? Justify with its angle property.



Self Assessment:

I understood all the concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood the all the concepts: ☐ Yes ☐ No

He/She need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment

Answers of Worksheet-1

Q.NO.	1	2	3	4	5	6	7	8	9	10
ANS.	B	B	C	B	B	C	B	B	C	C
Q.NO.	11	12	13	14	15	16	17	18	19	20
ANS.	C	B	C	B	C	C	B	B	C	A

Answers of Worksheet-2

Q.NO.	1	2	3	4	5
ANS.	65°	40°, 140°, 40°, 140°	13 cm	x = 25° A = 85° B = 95°	Trapezium
Q.NO.	6	7	8	9	10
ANS	PR = 9 cm SQ = 6.4 cm	TRUE	36°, 72°, 108°, 144°	A kite has 2 pairs of equal adjacent sides	12 cm
Q.NO.	11	12	13	14	15
ANS.	(a) x = 45° (b) 90°, 105°, 130°, 55° (c) Not a parallelogram	ΔABC is congruent to ΔCDA	(a)13 cm (b) 52 cm (c)120 cm²	a)900 m² b) ₹5,250	A kite can have two pairs of equal opposite angles without being a parallelogram
Q.NO.	16	17	18	19	20(i)
ANS.	a) y = 36° (b) 103°, 51°, 82°, 124°	Both AEFD and EBCF are parallelograms; each has area 60 cm².	(a) Trapezium (b) Area = 1,950 m² (c) Area ΔPRS = 580 m² (d) Matches		i)BCD = 70° ii)ABC = 110° Total fencing cost = ₹10,000
Q.NO.	20(ii)	21(i)	21(ii)		
ANS.	AC = 74 m; Area of ΔABD = 720 m²	Side = 12 cm; shorter diagonal = 18 cm; Area = 216 cm²	Total area = 32,400 cm²; Centre figure is a Square		

CHAPTER 5: NUMBER PLAY

POSTER 1 – NUMBER PLAY BASICS

5.1 IS THIS A MULTIPLE OF?

1. SUM OF CONSECUTIVE NUMBERS

Many natural numbers can be written as a sum of consecutive numbers in one or more ways.

Examples:

$7 = 3 + 4$

$10 = 1 + 2 + 3 + 4$

$12 = 3 + 4 + 5$

$15 = 7 + 8 = 4 + 5 + 6$

$= 1 + 2 + 3 + 4 + 5$

Key Ideas

- All odd numbers can be written as a sum of two consecutive numbers.
- All even numbers can be written as a sum of one or more consecutive numbers.
- 0 can be written as a sum of consecutive numbers using negative numbers.

2. FOUR CONSECUTIVE NUMBERS & SIGNS

Take 4 consecutive numbers (e.g., 3, 4, 5, 6). Place '+' and '−' between them. There are 8 possible expressions.

Numbers	Sign Pattern (+ for plus, − for minus)	Expression	Result
3 4 5 6	+ + +	3 + 4 + 5 + 6	18
3 4 5 6	+ + −	3 + 4 + 5 − 6	6
3 4 5 6	+ − +	3 + 4 − 5 + 6	8
3 4 5 6	+ − −	3 + 4 − 5 − 6	−4
3 4 5 6	− + +	3 − 4 + 5 + 6	10
3 4 5 6	− + −	3 − 4 + 5 − 6	−2
3 4 5 6	− − +	3 − 4 − 5 + 6	0
3 4 5 6	− − −	3 − 4 − 5 − 6	−12

➡ No matter how the '+' and '−' signs are placed between 4 consecutive numbers, the result is always an even number.

3. PARITY – WHY DO ALL 8 EXPRESSIONS HAVE THE SAME PARITY?

For any numbers a, b, c, d (not only consecutive), all expressions $a \pm b \pm c \pm d$ have the same parity (all even or all odd).

(1) Sign Switching

Switching the sign of any term changes the value by an even number.

Example:

$(a + b - c - d) - (a - b - c - d)$

$= 2b$ (even number)

➡ Numbers that differ by an even number have the same parity.

(2) Parity Rules

odd ± odd = even

even ± even = even

odd ± even = odd

So, $a \pm b$ has the same parity. Extending this, all $a \pm b \pm c \pm d$ have the same parity.

(3) Integer (Token) Model

Using positive (+) and negative (−) tokens: equal numbers of + and − tokens cancel or form pairs, leaving an even result.

➡ Net result is always even.

4. BREAKING EVEN – QUICK GUIDE

Identify which expressions are always even (for any integers).

Always Even

- ✓ $2a + 2b$
- ✓ $4m + 2n$
- ✓ $2u - 4v$
- ✓ $6m - 3n$
- ✓ $4k \times 3j$ (product of even and anything)
- ✓ 2×2 (i.e., 4)
- ✓ $\frac{2}{3} - \frac{3}{5}$ (sum of two even numbers)

Not Always Even

- ✗ $3g + 5h$
- ✗ $13k - 5k$
- ✗ b^2
- ✗ $x + 1$

5. KEY DEFINITIONS

- **Parity:** Whether a number is even or odd.
- **Even Number:** Divisible by 2.
- **Odd Number:** Not divisible by 2.
- **Consecutive Numbers:** Numbers that follow one after another in natural order (e.g., 3, 4, 5, 6).

Big Idea: Patterns in numbers help us make general conclusions without checking every case. Reasoning makes mathematics powerful and beautiful!

POSTER 2 – DIVISIBILITY MASTER CHART

5.2 CHECKING DIVISIBILITY QUICKLY

Divisibility Rules at a Glance

Divisible by	Rule / Trick	Example	Why it Works
2	Last digit is even (0, 2, 4, 6, 8).	4826 ✓ 1573 ✗	Even numbers end with an even digit.
3	Sum of digits is divisible by 3.	354 (3+5+4=12) ✓ 458 (4+5+8=17) ✗	Multiples of 3 have digit sum multiple of 3.
4	Last two digits form a number divisible by 4.	3168 (68 ✓) 5233 (33 ✗)	Based on place value (100 = 4 × 25).
5	Last digit is 0 or 5.	750 ✓ 8425 ✓ 731 ✗	Numbers ending in 0 or 5 are multiples of 5.
6	Divisible by both 2 and 3.	132 (✓ and ✓) 245 (✗ or ✗)	6 = 2 × 3
8	Last three digits form a number divisible by 8.	12056 (056=56 ✓) 35124 (124 ✗)	Based on place value (1000 = 8 × 125).
9	Sum of digits is divisible by 9.	729 (7+2+9=18 ✓) 456 (4+5+6=15 ✗)	Multiples of 9 have digit sum multiple of 9.
10	Last digit is 0.	640 ✓ 1230 ✓ 987 ✗	All multiples of 10 end with 0.
11	Difference of the sums of alternating digits is 0 or a multiple of 11.	5720 (5−7+2−0=0) ✓ 86134 (8−6+1−3+4=4) ✗	Based on alternating place values.
24	Divisible by both 3 and 8.	1248 (✓ and ✓) 2504 (✓ and ✓) 3766 (✗ or ✗)	24 = 3 × 8

Remainders and Divisibility

- If a number leaves remainder 0 when divided by n, it is divisible by n.
- If remainder ≠ 0, it is not divisible by n.
- Divisibility helps us check quickly without doing full division.

Always / Sometimes / Never

When we study a statement about numbers:

Always – True for all cases.

Sometimes – True for some cases.

Never – True for no cases.

Example: Statement: "The sum of two even numbers is odd."

Answer: Never (because even + even = even)

Factors & Multiples

- Factor of a number: a number that divides it exactly.
- Multiple of a number: obtained by multiplying it by whole numbers.
- Every number has infinitely many multiples but limited factors.

Example: Factors of 12 = 1, 2, 3, 4, 6, 12

Multiples of 12 = 12, 24, 36, 48, ...

Using LCM for Divisibility

If a number is divisible by two or more numbers, it is also divisible by their LCM.

Example: If a number is divisible by 4 and 6, it is divisible by LCM(4,6) = 12.

- Many divisibility rules come from place value.
- Look at last digit(s) or digit sum to decide.
- Reasoning helps us understand WHY the rule works.

POSTER 3 – DIGITAL ROOTS & DIGITS IN DISGUISE

DIGITAL ROOTS

Digital Root of a number: Repeatedly add the digits until you get a single-digit number. (For 0, the digital root is 0.)

How to Find

1. Add all the digits.
2. If the sum has more than one digit, add its digits again.
3. Repeat until you get a single digit.

Examples

- 9875 → 9+8+7+5=29 → 2+9=11 → 1+1=2 Digital root = 2
- 642 → 6+4+2=12 → 1+2=3 Digital root = 3
- 1001 → 1+0+0+1=2 Digital root = 2

Key Properties

- Numbers with the same digital root leave the same remainder when divided by 9.
- Digital root is 0 if the number is a multiple of 9.
- It is 3 if the number is not a multiple of 9 but is a multiple of 3.

Aryabhata's Method (for Divisibility by 9)

Subtract 1 from the number and add the digits of the result. Repeat the process. If you get 0 (or a multiple of 9), the number is divisible by 9.

Example: 8736

8736 − 1 = 8735 → 8+7+3+5 = 23

23 − 1 = 22 → 2+2 = 4

4 is not 0 or a multiple of 9 → 8736 is not divisible by 9.

5.3 DIGITS IN DISGUISE (CRYPTARITHMS)

A cryptarithm is a math puzzle where letters stand for digits (0–9). Each letter – has a fixed digit. Different letters represent different digits.

General Rules

- Each letter = one digit (0–9).
- Different letters = different digits.
- Leading letter of a number ≠ 0.
- Use logic, place value, and elimination.

Strategies to Solve

- ✓ Look for the leading letter (cannot be 0).
- ✓ Check the units column first.
- ✓ Find letters that produce a carry.
- ✓ Use uniqueness of digits to eliminate possibilities.

Addition Cryptarithm – Example

```
  S E N D
+ M O R E
---------
M O N E Y
```

Idea: Work column by column (from right to left).

- Y = (D + E) unit digit
- Find carry if D + E ≥ 10
- Continue to thousands place.

Multiplication Cryptarithm – Example

```
  T W O
x T W O
-------
  T W O
T W O
-------
T H R E E
```

Idea: Use place value and partial products carefully.

Practice Ideas

- ✓ Try simple puzzles first (2–3 letters).
- ✓ Use trial and error smartly.
- ✓ Cross out impossible options.
- ✓ Check your answer in the end.

Remember

Cryptarithms build logic, patience and number sense. Practice makes you a better problem solver!

Mathematics is full of patterns, rules and puzzles. Observe, reason and enjoy the beauty of numbers!

NUMBER PLAY

WORKSHEET-1

Section A: MCQs (Part-1) (10 × 1 = 10 marks)

1. Which of the following numbers is divisible by both 4 and 6? (Conceptual)

(a) 22 (b) 36 (c) 50 (d) 70

2. The LCM of two co-prime numbers 7 and 11 is: (Application)

(a) 7 (b) 11 (c) 77 (d) 1

3. The number of prime numbers between 10 and 30 is: (Logical Reasoning)

(a) 3 (b) 4 (c) 5 (d) 6

4. If the sum of digits of a number is 18, the number is divisible by: (Conceptual)

(a) 4 (b) 6 (c) 9 (d) 11

5. What is the HCF of 36 and 48? (Application)

(a) 6 (b) 12 (c) 18 (d) 24

6. A number is divisible by 8 if: (Conceptual)

(a) Its last digit is 8

(b) Its last two digits are divisible by 8

(c) Its last three digits form a number divisible by 8

(d) The sum of its digits is divisible by 8

7. Which of these is NOT a factor of 60? (Logical Reasoning)

(a) 12 (b) 15 (c) 16 (d) 20

8. The smallest prime number that is also even is: (Conceptual)

(a) 1 (b) 2 (c) 3 (d) 4

9. If LCM(a, b) = 120 and HCF(a, b) = 4, then a × b is equal to: (Application)

(a) 480 (b) 124 (c) 30 (d) 116

10. A magic square is filled with the numbers 1 to 9 such that the sum of every row, column and diagonal is equal. The magic sum is: (Logical Reasoning)

(a) 12 (b) 15 (c) 18 (d) 21

Section A: MCQs (Part-2) (10 × 1 = 10 marks)

11. Which of the following numbers is divisible by 3?

(a) 245 (b) 357 (c) 412 (d) 541

12. A number is divisible by 9 if:

(a) Its last digit is 9 (b) The sum of its digits is divisible by 9

(c) It is even (d) It ends with 0

13. Which of the following is a prime number?

(a) 27 (b) 39 (c) 41 (d) 51

14. The smallest composite number is:

(a) 1 (b) 2 (c) 3 (d) 4

15. Which number is divisible by both 2 and 5?

(a) 125 (b) 340 (c) 333 (d) 247

16. The sum of five consecutive integers is 145. What is the middle integer?

(a) 27 (b) 28 (c) 29 (d) 30

17. Without performing division, which of the following numbers is divisible by 9?

(a) 7,218 (b) 8,524 (c) 5,461 (d) 4,813

18. A number leaves a remainder of 4 when divided by 7. Which of the following numbers can be that number?

(a) 61 (b) 60 (c) 59 (d) 58

19. The digital root of a number is 9. Which of the following numbers satisfies this condition?

(a) 5,472 (b) 4,599 (c) 6,381 (d) 7,281

20. Using the divisibility rule of 11, which number is divisible by 11?

(a) 4,356 (b) 6,149 (c) 7,832 (d). 8,514

WORKSHEET-2

SECTION B SHORT ANSWER (11×2=20 MARKS)

1. Write all factors of 48 and count them.

2. State the divisibility rule for 11 and check whether 71,753 is divisible by 11.

3. Find the HCF of 56 and 98 using prime factorisation.

4. Are 21 and 22 co-prime? Give a reason.

5. Write the first five multiples of 17.

6. A number leaves remainder 0 when divided by 2, 3 and 5. What is the smallest such number greater than 0?

7. Express 180 as a product of its prime factors.

8. Identify all twin prime pairs between 1 and 50.

9. A shop owner has 84 red pens and 126 blue pens. He wants to pack them into boxes such that each box has the same number of pens of the same colour and no pen is left over. What is the maximum number of pens each box can have?

10. Using divisibility rules only, without actual division, check whether 7,392 is divisible by 4.

11. What is the difference between a 'prime number' and a 'composite number'? Give one example of each.

SECTION C SHORT ANSWER (6×3=18 MARKS)

12. Find the LCM of 18, 24 and 36 using prime factorisation. Also verify: LCM × HCF = Product of two numbers.

13. Two buses leave a bus stand at 6:00 AM. Bus A returns every 45 minutes and Bus B returns every 60 minutes. At what time will they both be at the bus stand together again?

14. List all prime numbers up to 50. How many prime numbers are there between 1 and 50?

15. A number N is such that: N is a 3-digit number, N is divisible by 9, the sum of its digits is 18, and it is also divisible by 6. Find at least two such values of N.

16. The length and breadth of a rectangular park are 90 m and 75 m respectively. What is the minimum length of rope needed to divide the park into square plots of equal size, leaving no area uncovered? (Find side of each square plot.)

17. Riya says, 'The product of two consecutive even numbers is always divisible by 8.' Is she correct? Verify with two examples and give a mathematical reason.

18.A school organises a sports day. There are 180 boys and 252 girls. The principal wants to form groups such that each group has equal numbers and consists of either all boys or all girls.

(a) What is the maximum size of each group?

(b) How many groups of boys and girls will be formed?

(c) What mathematical concept did you use? Explain it briefly.

19. A digital clock shows time in hours (H) and minutes (M). The minute hand completes one round every 60 minutes. Arjun notices that the seconds hand, minute hand and hour hand of his clock align at 12:00:00.

(a) After how many seconds will the seconds hand and minute hand align again?

(b) After how many minutes will the minute hand and hour hand align for the first time after 12:00?

(c) Explain how LCM and HCF are connected to such real-life periodic events.

20. Study the following patterns and answer the questions:

Pattern 1: $1 + 3 = 4 = 2^2$, $1 + 3 + 5 = 9 = 3^2$, $1 + 3 + 5 + 7 = 16 = 4^2$

Pattern 2: $6 = 1 + 2 + 3$, $28 = 1 + 2 + 4 + 7 + 14$, 496 = sum of all its proper factors

(a) Extend Pattern 1 to write the next two steps.

(b) Numbers in Pattern 2 are called _____ numbers. Name two more such numbers.

(c) Using Pattern 1, find the sum of first 10 odd numbers without adding them individually.

(d) A number is called 'deficient' if the sum of its proper factors is less than the number. Give two examples.

Case Study Based Questions (4 × 2 = 8 marks)

21. The school garden is rectangular, measuring 126 m in length and 84 m in breadth. The gardener wants to plant trees at equal intervals along the boundary of the garden (at corners too), using the least number of trees possible. He also wants to divide the garden into square plots of maximum possible size for different flowers

(i) What is the maximum side length of each square plot? Show working.

(ii) How many square plots will the garden be divided into? Justify.

(iii) At what intervals should trees be planted along the boundary?

(iv) How many trees are needed in total to go around the boundary at those intervals?

22. Three lighthouses on a coast flash their lights at regular intervals. The table below shows their flashing schedule:

Lighthouse	Colour of Flash	Flashing Interval
Lighthouse A	Red	12 seconds
Lighthouse B	Green	18 seconds
Lighthouse C	Blue	24 seconds

(i) All three lighthouses flash together at 8:00:00 PM. After how many seconds will they flash together again?

(ii) Find the LCM of 12, 18 and 24 using prime factorisation. Show all steps.

(iii) How many times will Lighthouse A and Lighthouse B flash together (not C) in the first 3 minutes after 8:00:00 PM?

(iv) A sailor can only see lights flashing simultaneously from at least two lighthouses. How many such simultaneous flashes (of any two or all three) will he observe in the first 5 minutes?

Self Assessment:

I understood all concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood the all the concepts ☐ Yes ☐ No

He/She need more practice ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment:

Answers of Worksheet-1:

Q	1	2	3	4	5	6	7	8	9	10
Ans	b	c	c	c	b	c	c	b	a	b
Q	11	12	13	14	15	16	17	18	19	20
Ans	b	b	c	d	b	c	a	b	a	d

Answers of Worksheet:

Q	1	2	3	4	5
Ans	1,2,3,4,6,8,12, 16,24,48 total 10 factors	yes	14	YES	17,34,51, 68,85
Q	6	7	8	9	10
Ans	LCM=30	$2^{2\times}32\times5$	(3,5), (5,7), (11,13)	42	YES
Q	11	12	13	14	15
Ans	prime exactly two factor coprime: more than two factor.	HCF =6 LCM=72	LCM=180 minutes 9:00 am	15	990, 882
Q	16	17	18	19	20
Ans	30 plots	Riya is correct	36 students Boys =5 groups Girls=7 groups	a)every 60 sec. b)first alignment 1:05:27 am c) HCF and LCM explain	A) 5^2,6^2
Q	21	22			
Ans	A)42M b) 6plots c)42 m d) preimeter =420 m trees=10	LCM=72 Sec. Lcm =72 iii) 36 sec. in 3 min. =180 sec. iv) 16			

CHAPTER 6: WE DISTRIBUTE, YET THINGS MULTIPLY

SUMMARY OF CONCEPTS & DEFINITIONS

1. BIG IDEA

Algebra uses letters to write general statements about patterns and relations.

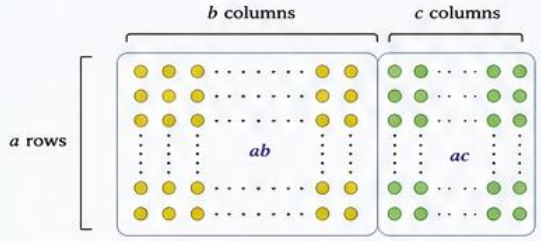
Distributivity is a property that relates multiplication and addition. It helps us prove statements, discover patterns and solve problems.

2. DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION

$$a\,(b + c) = ab + ac$$

This means multiplying a by $(b + c)$ is the same as multiplying a by b and a by c, then adding.

Diagram (Area Model)



Also,

$$(a + b)\,c = ca + cb = ac + bc$$

(Distributivity + Commutativity)

Remember: $a\,(b + c)$ and $23\,(27 + 1)$ mean $a \times (b + c)$ and $23 \times (27 + 1)$ respectively.

3. INCREMENTS IN PRODUCTS

Let the initial two numbers be a and b (product $= ab$).

1. If second number b is increased by 1:

$$a\,(b + 1) = ab + a$$

Increase $= a$

Example: $23\,(27 + 1) = 23 \times 27 + 23$; Increase $= 23$

2. If first number a is increased by 1:

$$(a + 1)\,b = ab + b$$

Increase $= b$

Example: $(23 + 1)\,27 = 23 \times 27 + 27$; Increase $= 27$

3. If both numbers are increased by 1:

$$(a + 1)\,(b + 1) = ab + (a + b + 1)$$

Increase $= a + b + 1$

Example: $(23 + 1)\,(27 + 1) = 23 \times 27 + (23 + 27 + 1)$; Increase $= 51$

4. If one number is increased by 1 and the other decreased by 1:

$$(a + 1)\,(b - 1) = ab + b - a - 1$$

Increase $= b - a - 1$

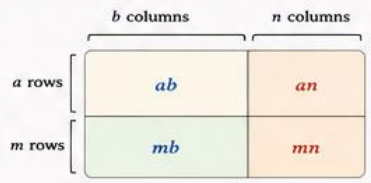
Example: $(23 + 1)\,(27 - 1) = 23 \times 27 + 27 - 23 - 1$; Increase $= 3$

General Case (Increase by m and n)

If a is increased by m and b by n:

$$(a + m)(b + n) = ab + mb + an + mn$$

Increase $= an + bm + mn$



$$(a + m)\,(b + n) = ab + mb + an + mn \quad \text{(Identity 1)}$$

4. KEY DEFINITIONS

- **Distributive Property:** $a\,(b + c) = ab + ac$ and $(a + b)c = ac + bc$.
- **Product:** The result of multiplying two numbers.
- **Increase in Product:** The amount by which the product becomes greater after a change.
- **Identity:** An equality that is true for all values of the letter-numbers (or all numbers in a certain set).
- **Commutative Property:** $ab = ba$.

5. WHAT TO REMEMBER (AT A GLANCE)

Situation	New Product	Increase in Product
Only b increased by 1	$a\,(b + 1)$	a
Only a increased by 1	$(a + 1)\,b$	b
Both a and b increased by 1	$(a + 1)\,(b + 1)$	$a + b + 1$
a increased by 1, b decreased by 1	$(a + 1)\,(b - 1)$	$b - a - 1$
a increased by m, b increased by n	$(a + m)\,(b + n)$	$an + bm + mn$

6. WORKS FOR ALL INTEGERS

These results hold even when a and b are negative integers.

Example:

Let $a = -5$, $b = 8$

Both increased by 1:

$(a + 1)(b + 1) = (-4)(9) = -36$

$ab + (a + b + 1) = (-5)(8) + (-5 + 8 + 1) = -40 + 4 = -36$ ✓

One increased, one decreased:

$(a + 1)(b - 1) = (-4)(7) = -28$

$ab + b - a - 1 = (-5)(8) + 8 - (-5) - 1 = -40 + 8 + 5 - 1 = -28$ ✓

Integers also satisfy the distributive property: $x(y + z) = xy + xz$ (for any integers x, y, z). Hence all the above formulas are valid for integers.

7. VISUAL SUMMARY

Think: Break a product into smaller parts using distributivity, then combine.

8. QUICK CHECK

Find the increase in the product in each case (in terms of a and b).

1. $a\,(b + 1)$
2. $(a + 1)\,b$
3. $(a + 1)\,(b + 1)$
4. $(a + 1)\,(b - 1)$
5. $(a + m)\,(b + n)$

CORE MESSAGE: Distribute, simplify, and patterns appear. Distributivity turns complex expressions into simple truths!

WORKSHEET-1

Section A: MCQs (Part-1) (10 × 1 = 10 marks)

1. Using the distributive property, 5 × (8 + 2) =

a) 40 b) 50 c) 45 d) 35

2. Which expression is equivalent to 7 × (10 + 3)?

a) 7 × 10 + 7 × 3 b) 7 × 10 − 7 × 3 c) 10 × 3 × 7 d) 7 + 10 + 3

3. The value of 9 × (6 − 2) is:

a) 54 b) 36 c) 32 d) 28

4. Using the distributive property, 12 × 11 can be written as:

a) 12 × (10 + 1) b) 12 + 11 c) 12 × (10 − 1) d) 11 × (11 + 1)

5. What is the value of 4 × (20 + 5)?

a) 80 b) 90 c) 100 d) 120

6. Which property is used in 8 × (3 + 4) = (8 × 3) + (8 × 4)?

a) Commutative Property b) Associative Property

c) Distributive Property d) Closure Property

7. Find the value of 15 × (4 + 2).

a) 60 b) 75 c) 90 d) 120

8. 6 × (9 − 4) =

a) 54 b) 30 c) 24 d) 18

9. Which of the following is correct?

a) 5 × (7 + 2) = 5 × 7 + 5 × 2 b) 5 × (7 + 2) = 5 + 7 + 2

c) 5 × (7 + 2) = 5 × 7 × 2 d) 5 × (7 + 2) = 7 + 2

10. The value of 13 × (5 + 5) is:

a) 65 b) 100 c) 130 d) 150

MCQs (Part-2) (10 × 1 = 10 marks)

11. Which of the following best describes the Distributive Law?

(A) $a \times (b + c) = (a \times b) + c$ (B) $a \times (b + c) = (a \times b) + (a \times c)$

(C) $a + (b \times c) = (a + b) \times (a + c)$ (D) $a \times (b + c) = a \times b \times c$

12. Using the distributive property, 7×99 equals:

(A) $700 – 7$ (B) $7 \times 100 – 7$ (C) $7 \times 100 + 7$ (D) $700 + 99$

13. Which identity is used when we write $15 \times 12 = 15 \times (10 + 2)$?

(A) Commutative property (B) Associative property

(C) Distributive property (D) Identity property

14. The value of 25×102 calculated using distribution is:

(A) 2,500 (B) 2,450 (C) 2,550 (D) 2,525

15. If $a = 6, b = 4, c = 3$, then $a \times (b - c)$ equals:

(A) 18 (B) 6 (C) 9 (D) 24

16. Which of the following shows an incorrect application of distributive law?

(A) $3 \times (4 + 5) = 12 + 15$ (B) $5 \times (6 - 2) = 30 - 10$

(C) $4 \times (3 \times 2) = 12 + 8$ (D) $7 \times (10 + 1) = 70 + 7$

17. 48×25 can be simplified as:

(A) $48 \times 20 + 48 \times 5$ (B) $48 \times 20 \times 5$ (C) $50 \times 25 – 2$ (D) $48 + 25 \times 4$

18. An orchard has 36 rows of apple trees and 36 rows of mango trees, each row having 15 trees. Total trees = ?

(A) 1080 (B) 1440 (C) 720 (D) 900

19. The expression $(a + b) \times c$ is equivalent to:

(A) $a + bc$ (B) $ac + bc$ (C) $ac + b$ (D) $a + b + c$

20. How does the distributive law help in mental arithmetic?

(A) It changes the order of numbers

(B) It breaks hard multiplications into simpler ones

(C) It converts multiplication into addition only

(D) It always gives approximate answers

WORKSHEET-2

SECTION B SHORT ANSWER (11×2=20 MARKS)

1. Using the distributive law, evaluate 8 × 103 without a calculator. Show one step.

2. Fill in the blank using distributive law and state the property used:

34 × 18 = 34 × _____ + 34 × _____

3. Verify: 6 × (9 − 4) = 6 × 9 − 6 × 4. Show both sides separately.

4. A teacher distributes 12 chocolates to each of 24 boys and 16 girls. Using distribution, find the total chocolates distributed.

5. Write distributive law over subtraction for whole numbers with an example.

6. Using distribution, find the value of 99 × 13.

7. State whether True or False and justify: 5 × (3 + 2) = (5 × 3) + 2.

8. Simplify using distribution: 15 × 8 + 15 × 2.

9. If p = 11, q = 5, r = 3, verify: p × (q + r) = p × q + p × r.

10. A vegetable vendor sells carrots at ₹18 per kg. He sells 6 kg on Monday and 9 kg on Tuesday. Using distribution, find his total earnings.

SECTION C SHORT ANSWER (6×3=18 MARKS)

11. A shopkeeper buys 25 boxes. Each box has 12 red pens and 8 blue pens. Using the distributive law, find: (i) Total pens in all boxes. (ii) Which law saves calculation steps here?

12. Two numbers are in the ratio 3 : 5. If their sum is 48, find both numbers. Then multiply the smaller number by 11 using distribution over (10 + 1). Show all steps.

13. Explain with two different numerical examples why multiplication distributes over both addition and subtraction. Also explain why it does NOT distribute in the same way over multiplication (e.g., $a \times (b \times c) \neq (a \times b) \times (a \times c)$).

14. A rectangle has length (x + 4) cm and breadth 6 cm. Find its area using the distributive law. If x = 5, find the numeric value of the area.

15. Riya has 3 times as many stickers as Meera. Meera has (p + q) stickers. Write an expression for Riya's stickers and expand it using distribution. If p = 7 and q = 3, find the total stickers both have together.

16. Using distribution, show that 999 × 999 = (1000 − 1) × (1000 − 1). Apply the distributive law step-by-step to arrive at the answer. (Hint: treat this as $(a - b)(a - b) = a^2 - 2ab + b^2$).

Section D: Long Answer (3 × 5 = 15 marks)

17. A school organises a Sports Day. There are 18 boys and 22 girls in each of 15 sections. (a) Using distributive law, find the total number of boys in the school. (b) Using distributive law, find the total number of girls in the school. (c) Using the distributive law again, find the combined total of students in all sections. (d) If each student is given a kit worth ₹45, find the total expenditure using distribution.

18. Prove the distributive law $a \times (b + c) = a \times b + a \times c$ for $a = 14$, $b = 7$, $c = 3$.

Also prove $a \times (b - c) = a \times b - a \times c$ for the same values. Explain in your own words why this law is useful in real-life calculations. Give one real-life scenario where you would apply this law.

19. A tile layer lays tiles in a hall of length 36 m and breadth (12 + 8) m. (a) Use the distributive law to find the total area of the hall. (b) If tiles of size 1 m × 1 m are used and each tile costs ₹55, find the total cost using distribution. (c) If 10% tiles are to be kept as spare, how many extra tiles are needed? (Use distribution where possible.)

SECTION E CASE BASED (2×4=8 MARKS)

20. **The School Staionary Store**

The school stationery store stocks notebooks, pens, and geometry boxes. Every week, the store manager orders supplies. This week, she orders 35 packets of notebooks. Each packet contains 6 single-line notebooks and 4 four-line notebooks. She also orders 35 geometry boxes at ₹85 each and 35 pen sets at ₹55 each.

(i) Using the distributive law, find the total number of notebooks ordered this week. Clearly state which form of the law you used.

(ii) The manager wants to find the total money spent on geometry boxes and pen sets together. Using the distributive law, calculate the total amount and explain how distribution saved calculation effort here.

21. **The Community Garden**

A community has built a rectangular garden with length 48 m. The breadth is divided into two parts: 13 m is used for flowering plants and 7 m is used for vegetable beds. The municipality decides to lay a cement path of width 1 m all around the garden. The cost of cementing is ₹120 per sq. m.

(i) Using the distributive law, find the total area of the garden (excluding the path). Show all steps clearly.

(ii) If the total area including the path is 1100 sq. m, find the area of the path alone. Then, using distribution, find the total cost of cementing the path at ₹120 per sq. m, writing ₹120 as ₹(100 + 20).

Self Assessment:

I understood all the concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood all the concepts: ☐ Yes ☐ No

He/She need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment

Answers Worksheet-1

Q	1	2	3	4	5	6	7	8	9	10
ANS	B	A	B	A	C	C	C	B	A	C
Q	11	12	13	14	15	16	17	18	19	20
ANS	B	B	C	C	B	C	A	A	B	B

Answers Worksheet-2

Q	1	2	3	4	5
ANS	824	612	**LHS =RHS**	480 chocolates	**25**
Q	6	7	8	9	10
ANS	1287	FALSE	150	LHS = RHS	270
Q	11	12	13	14	15
ANS	500 pens	198	Distribution does NOT hold over multiplication	54sq cm	40 stickers
Q	16	17	18	19	20
ANS	998,001	27,000	2760.	**A)** 720 sq m B)39,600 C)72 tiles	A)350 B)4,900
Q	21				
ANS	A)960 sq m B)16,800				

7 CHAPTER 7: PROPORTIONAL REASONING – 1

SUMMARY OF CONCEPTS & DEFINITIONS

1. OBSERVING SIMILARITY IN CHANGE

Digital images can be resized and oriented.
We observe how changes in width and height affect the look of an image.

Set of Images


Image A


Image B


Image C


Image D


Image E

Measurements (in mm)

Image	Width	Height
Image A	60	40
Image B	40	20
Image C	30	20
Image D	90	60
Image E	60	60

- **Images A, C and D look similar.** Their widths and heights change by the same factor (through multiplication).
- **Images B and E look different.** Their widths and heights do not change by the same factor.

If width and height change by the same factor, the images look similar. Such changes are proportional.

2. RATIOS

A ratio compares two quantities of the same kind.
It is written as $a : b$ and read as "a is to b".

Key Points

- a and b are the terms of the ratio.
- In $a : b$, for every 'a' units of the first quantity, there are 'b' units of the second quantity.
- Ratio of width : height
  - Image A = 60 : 40
  - Image C = 30 : 20
  - Image D = 90 : 60

The ratios of width to height of images A, C and D are proportional. (Their terms change by the same factor.)

3. RATIOS IN THEIR SIMPLEST FORM

Two ratios are in proportion if their simplest forms are the same.
To get the simplest form → divide both terms by their HCF.

Image	Ratio (W : H)	HCF	Simplest Form
A	60 : 40	20	3 : 2
B	40 : 20	20	2 : 1
C	30 : 20	10	3 : 2
D	90 : 60	30	3 : 2
E	60 : 60	60	1 : 1

Images A, C and D have the same simplest form (3 : 2). So, their ratios are proportional.

Images B and E have different simplest forms, so their ratios are not proportional to A, C and D.

We use $a :: b :: c : d$ to show that the ratios $a : b$ and $c : d$ are proportional.

Examples: • 60 : 40 :: 30 : 20 • 60 : 40 :: 90 : 60

4. INCREMENTS IN PRODUCTS (USING DISTRIBUTIVITY)

Let the initial two numbers be a and b (product = ab).
Using distributive property:

$$a(b + c) = ab + ac$$
$$(a + b)c = ac + bc$$

1. If second number b is increased by 1:
   $a(b + 1) = ab + a$
   Increase = a
2. If first number a is increased by 1:
   $(a + 1)b = ab + b$
   Increase = b
3. If both a and b are increased by 1:
   $(a + 1)(b + 1)$
   $= ab + (a + b + 1)$
   Increase = $a + b + 1$
4. If a is increased by 1 and b is decreased by 1:
   $(a + 1)(b - 1)$
   $= ab + b - a - 1$
   Increase = $b - a - 1$

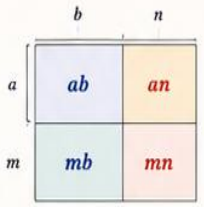
General Case

If a is increased by m and b by n:

$(a + m)(b + n)$
$= ab + mb + an + mn$

Increase = $an + bm + mn$

(Identity 1)



Product = sum of products of each term of $(a + m)$ with each term of $(b + n)$.

5. PROBLEM SOLVING WITH PROPORTIONAL REASONING

Example 1

Are 3 : 4 and 72 : 96 proportional?

3 : 4 is in simplest form.
HCF of 72 and 96 = 24.
72 : 96 = 3 : 4

➡ Both ratios are same in simplest form. Hence, they are proportional.

Example 2

6 glasses of lemonade need 10 spoons of sugar.
How many spoons for 18 more glasses (same sweetness)?

Ratio of glasses : spoons = 6 : 10
For 18 more glasses → 18 : x
Since ratios are proportional:

$$6 : 10 = 18 : x \Rightarrow x = \frac{18 \times 10}{6} = 30$$

➡ She should add 30 spoons of sugar.

KEY TAKEAWAYS

- ✓ Similarity comes from proportional change in dimensions.
- ✓ Ratio compares two quantities.
- ✓ Simplify ratios using HCF.
- ✓ Equal simplest forms ⇒ Ratios are proportional.
- ✓ Distributive property helps find changes in products.
- ✓ Proportional reasoning helps solve real-life problems.

CORE MESSAGE: When quantities change in the same proportion, relationships stay the same.
Ratios and proportional reasoning help us understand patterns and solve everyday problems!

PROPORTIONAL REASONING-I

WORKSHEET-1

Section A: MCQs(PART –I) (10 × 1 = 10 marks)

1.A Ratio is a form of comparison by _____.

(a) Addition (b) Subtraction (c) Multiplication (d) Division

2.If 5 notebooks cost ₹ 80, then the cost of 1 notebook is:

(a) ₹15 (b) ₹20 (c) ₹ 16 (d) ₹30

3.If a ratio Expressed in Lowest form has no common factor other __ In its terms

(a) zero (b) 1 (c) 10 (d) HCF

4.If 8 kg of rice costs ₹480, then 12 kg of rice will cost:

(a) ₹600 (b) ₹720 (c) ₹760 (d) ₹840

5.The ratio of 15 to 25 in simplest form is:

(a) 3 : 5 (b) 1 : 5 (c) 15 : 25 (d) 25 : 15

6.If x:12=5:6 then the value of x is:

(a) 8 (b) 10 (c) 12 (d) 15

7. The HCF of 72 and 96?

(a) 8 (b) 9 (c) 12 (d) 24

8.Which two ratios are equal to 4:11 ?

(a) 18:20 (b) 16:20 (c) 12 :30 (d) 24: 66

9. 19 : 76 = 9: ___

(a) 36 (b) 9 (c) 27 (d) 18

10. The Ratio of side of a square and its perimeter is always

(a) 1 : 4 (b) 3 : 1 (c) 4 : 1 (d) none of these

Section A: MCQs(PART –2) (10 × 1 = 10 marks)

11. A school map is enlarged so that every length becomes three times its original size. What happens to the ratio of length to breadth of every rectangular classroom?

(a) It becomes three times. (b) It becomes one-third.
(c) It remains unchanged. (d) It doubles.

12. A recipe uses flour and sugar in the ratio 5 : 2. If 700 g of flour is used, how much sugar is required?

(a) 240 g (b) 260 g (c) 280 g (d) 300 g

13. Which pair of ratios is proportional?

(a) 15 : 20 and 9 : 12 (b) 8 : 15 and 12 : 25 (c) 7 : 9 and 14 : 16 (d) 5 : 12 and 15 : 40

14. A farmer divides 84 mangoes between two children in the ratio 4 : 3. How many mangoes does the child with the larger share receive?

(a) 42 (b) 48 (c) 52 (d) 56

15. A car covers 180 km using 15 litres of petrol. At the same efficiency, how much petrol will be needed to travel 300 km?

(a) 20 litres (b) 22 litres (c) 24 litres (d) 25 litres

16. A mixture contains red and blue beads in the ratio 3 : 5. If there are 80 beads altogether, how many are blue?

(a) 25 (b) 30 (c) 40 (d) 50

17. A bottle contains 2 litres of juice. How many millilitres of juice does it contain?

(a) 200 mL (b) 500 mL (c) 1000 mL (d) 2000 mL

18. A photograph has dimensions 24 cm × 16 cm. Which of the following dimensions will produce a similar photograph without distortion?

(a) 30 cm × 24 cm (b) 36 cm × 24 cm (c) 30 cm × 18 cm (d) 42 cm × 30 cm

19. A shopkeeper packs rice in bags weighing 2 kg each. If 15 such bags are prepared, what is the total weight of rice?

(a) 25 kg (b) 28 kg (c) 30 kg (d) 32 kg

20. A ribbon 72 m long is cut into two pieces in the ratio 5 : 4. What is the length of the longer piece?

(a) 32 m (b) 36 m (c) 40 m (d) 45 m

WORKSHEET-2

Section B: Short Answer (10 × 2 = 20 marks)

1.Length of the rectangle is 30 metre And the breadth is 25 metre, Write the ratio between Its length and breadth**.**

2. Write the ratio of 3 m to 480 cm in simplest form.

3. In a school, there are 5 teachers and 150 students. The ratio of teachers to students in school is 5 : 150. What will be the number of teachers if there are 1500 students in school?

4. If 9 kg sugar costs ₹405, find the cost of 10 kg sugar.

5. A shopkeeper sells 5 kg of apples for ₹350. Find the cost of 8 kg apples.

6. 36 : 81 = x : 63 find x

7. Mathematics book of class 8 (part 1) has total 180 pages And the chapter proportional reasoning (part one runs from page number 159 to 218. What is the ratio between Number of pages of this chapter and total number of pages in the book.

8. Are these ratios equal 13: 39 and 27: 84?

9. A train travels 240 km in 4 hours. How far will it travel in 7 hours at the same speed?

10. Find the missing number: 63 :105 = 39 : x

Section C: Short Answer (6 × 3 = 18 marks)

11. The cost of 12 identical chairs is ₹9,600. Find the cost of 18 such chairs.

12. There are 102 teachers in a school of 3300 Students find the ratio of -- (a) number of teachers to the number of students (b) number of students to the total number of staff including 1 principal,1 vice principal and 6 office employees.

13. In a recipe, 4 cups of flour and one cup of sugar are needed for 16 cookies. How many cups of flour and sugar are needed for 28 such cookies?

14.A cyclist covers 90 km in 6 hours.

(a) How many hours will he take to cover 150 km at the same speed?

(b) how much he covers in total 13 hours if he takes rest of 2 hours and moves with same speed.

15. Divide 72 Chocolates In the ratio 5:4.

16. State TRUE or FALSE (a) 35:70::2:4 (b) 7:12 :: 9 : 27

Section D: Long Answer (4 × 5 = 20 marks)

17.A factory produces 450 bottles in 6 hours.

(a) How many bottles are produced in 10 hours? (b) How many bottles are produced in 15 hours?

18. The ratio of boys to girls in a class is 3 : 5. If there are 15 boys find the number of girls. Find the total number of students in the class. Also find ratio of total students to the number of boys.

19. A farmer uses 28 kg of seeds for sowing 14 acres of land.

a. How many kilograms of seeds will be required for 18 acres?

b. How many acres land can be seeded with 63.5 kilograms seeds?

20. A rectangle has length and width 12 m and 9 m respectively. What is the ratio between-

(a) length : perimeter

(b) perimeter : area

Case Study Based (2 × 4 = 8 marks)

21.Raman purchases 10 pens for ₹ 160 and Manan buys 6 pens for ₹ 84. Find the ratio Of number of Pens to the cost of pens In both cases . Also write whether these two Ratios are Proportional or not?

22.An electric pole, 14 metres high, casts a shadow of 10 metres long.

(a) Find the height of a tree that casts a shadow of 15 metres under similar conditions.

(b) If the height of a tower near the tree is 28 m what is the length of its shadow of 15 metres under similar conditions.

Self Assessment:

I understood all the concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

I understood all the concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment

Answers of Worksheet-1:

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans	d	c	b	b	a	b	d	d	a	a
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans	c	c	a	b	d	d	d	b	c	c

Answers of Worksheet-2:

Q.No.	1	2	3	4	5
Ans	6:5	5:8	50	450	560
Q.No.	6	7	8	9	10
Ans	28	11:90	1:3 9:24; no	420 km	65
Q.No.	11	12	13	14	15
Ans	14,400	a. 17 : 550 b. 30:1	7; 1.75	10 hrs, 165 km	40 and 32
Q.No.	16	17	18	19	20
Ans	True false	750 , 1125 bottles	25 girls 8:3	36 kg; 31.75 acres	2:7; 7: 18
Q.No.	21	22			
Ans	1:16; 1:14; No	21 m; 20 m			